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Influence of axisymmetric flow structure on energy dissipation in a ring-shaped layer of magnetic fluid

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Abstract

Numerical study is made of 3D-flow structure and energy dissipation in the bounded ring-shaped layer of a magnetic fluid in the gap between two rotating coaxial solids of complicated shape. Influence of transverse flow on energy dissipation increases with growing of a rotation speed. This problem is important, for example, for analysis of high speed magnetofluid seals.

1. Introduction

The subject of present study is the flow in a ring-shaped bounded volume of a magnetic fluid between two co-axial solids of complicated shape, with the inner one rotating. As the shape of a solid surface is not a circular cylinder, there is not only the azimuthal component v_φ of flow velocity. Radial v_r and axial v_z components of the flow velocity exist even at a very small Taylor number. At small Taylor number the azimuthal component of the velocity is much more than others, and energy dissipation depends on this component only. As a result, the energy dissipation $\dot{E}$ is proportional to U^2 for constant properties of magnetic fluid, where U is the linear velocity of the inner solid surface. With increasing the Taylor number, the flow structure becomes more complicated, the components of the velocity v_r and v_z become comparable with v_φ and the ratio $\dot{E}/U^2$ declines from the constant.

The above-mentioned problem is similar to the flow in magnetic fluid seals and may be useful for analyzing a temperature regime of such kind of devices. In previous investigations [1–3], the assumption has been made on unidirectional motion of a magnetic fluid. The present study takes into account all components of the velocity.

2. Governing equations

A study is made of the 3D flow of a viscous incompressible magnetic fluid in the gap between the coaxial solids of complicated shape (Fig. 1a). The inner solid surface is rotating with linear velocity U and the outer one is motionless. Although the shape of a free surface of a magnetic fluid depends on the velocity of the inner rotating surface [4], the assumption is made of a high-gradient magnetic field and, as a result, of a constant shape of free surface. This shape is given as a part of a circle.

Consideration is made of energy dissipation in the layer of a magnetic fluid. Dissipation is governed by the velocity distribution in the volume of fluid and, in general tensor form, could be written as

$$\dot{E} = \frac{1}{2}\eta \int_\Omega \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right)^2 \mathrm{d}\Omega, \qquad (1)$$

where Ω is the volume of fluid, η is the fluid dynamic viscosity, $i = 1, 2, 3$.

As surfaces are non-parallel, axisymmetrical flow is three-dimensional, i.e. all components of velocity v_r, v_z and v_φ are not equal to zero in cylindrical coordinate system. Axial symmetry gives the opportunity to solve the equations of motion in the plane $\varphi = \text{const}$. For components of velocity v_r and v_z it is possible to define the stream function ψ and vorticity ω in the following way:

$$\omega = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}, \quad v_r = -\frac{1}{r}\frac{\partial \psi}{\partial z}, \quad v_z = \frac{1}{r}\frac{\partial \psi}{\partial r}. \qquad (2)$$

This Navier–Stokes equation could be rewritten in the following dimensionless form:

$$\mathrm{Re}\left[-\frac{1}{r^2}\frac{\partial \psi}{\partial z}\frac{\partial (r v_\varphi)}{\partial r} + \frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial v_\varphi}{\partial z} \right] = \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (r v_\varphi)}{\partial r} \right] + \frac{\partial^2 v_\varphi}{\partial z^2}, \qquad (3)$$

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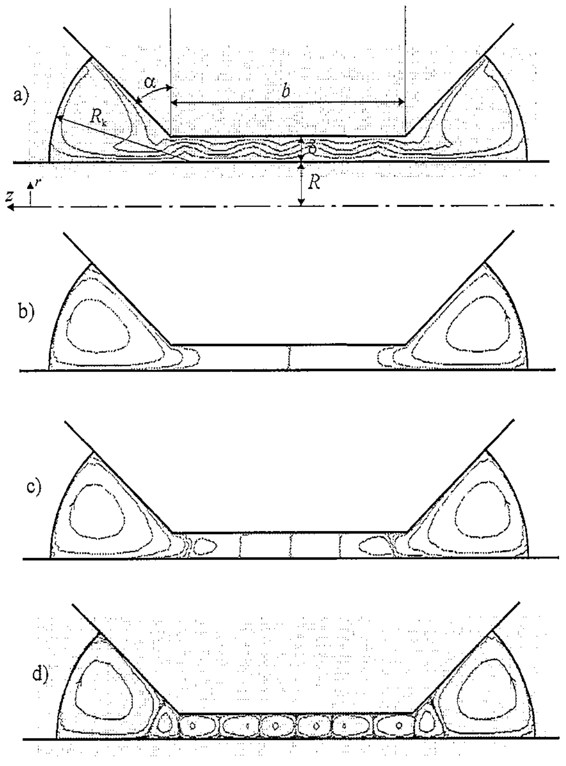

Fig. 1. Problem geometry and flow patterns. $R = 10$, $\delta = 0.2$, $\alpha = 45°$, $b = 2$, $R_k = 1.22$. (a) Isochores of azimuthal velocity v_φ, Ta = 57. Sketch of streamlines with (b) Ta = 2.8, (c) Ta = 21, (d) Ta = 57.

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\omega)\right]+\frac{\partial^2\omega}{\partial z^2}+\mathrm{Re}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial z}\omega\right)-\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\omega\right)\right]=-2\,\mathrm{Re}\frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial z}, \tag{4}$$

$$\frac{1}{r}\frac{\partial^2\psi}{\partial z^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right)=-\omega, \tag{5}$$

where $\mathrm{Re} = Ul/\nu$ is the Reynolds number, $l = 1$ mm is the characteristic value of dimension, ν is the fluid kinematic viscosity.

No-slip conditions are assigned on the solid surfaces, and shear stresses are equal to zero on the free surfaces, the stream function ψ is equal to zero on all boundaries.

Eqs. (3)–(5) with boundary conditions are solved by the control-volume finite-element method [5]. The applied method uses the exponential interpolation functions for vorticity ω and azimuthal velocity component v_φ and linear interpolation function for stream function ψ on each triangle element. The vorticity ω on the boundaries is calculated by the method described in Ref. [6] with use of boundary conditions.

3. Results

A computer simulation has been done for the following range of parameters: Re = 0–2500, $\delta = 0.2$, $b = 0$–2. Fig. 1 shows flow patterns in the cross-section φ = const. for one set of parameters. It is seen that the fluid flow in the plane r, z exists for any Taylor number $\mathrm{Ta} = \mathrm{Re}\ \delta\sqrt{\delta/R}$, even for a very small value (Fig. 1b, Ta = 2.8). The direction of fluid motion along the free surface is from inner to outer solid. Secondary cells of small intensity arise in the narrow part of the gap with increasing Taylor number. At small Taylor number (Fig. 1c, Ta = 21) intensity of the motion in the secondary cells is much less than of the primary one. For supercritical values of the Taylor number (Ta > 41.3) the fluid motion has almost equal intensity in all cells in the cylindrical part of the gap (Fig. 1d, Ta = 57), and it is possible to say that these cells are equivalents of Taylor cells for unstable cylindrical Kuette flow. The number of the cells depends on the gap geometry and rotation speed of the inner solid boundary.

The azimuthal component of the velocity, v_φ, gives the main contribution to energy dissipation in the ring-shaped layer. The influence of the transverse components v_r and v_z on the energy dissipation becomes essential with increasing Taylor number due to changing flow structure in the gap and the redistribution of v_φ. Fig. 2 shows that the ratio $\dot{E}/\eta l U^2$ due to the complicated structure of the flow differs from constant and increases rapidly at Ta ~ 5. Especially, hard difference can be seen at supercritical Taylor numbers when the motion in the cells in the gap is developed.

4. Conclusions

It is shown that the energy dissipation magnitude is determined by the velocity field distribution in a magnetic fluid volume. Influence of transverse flow on energy dissipation becomes essential with increasing rotation speed of the inner solid.

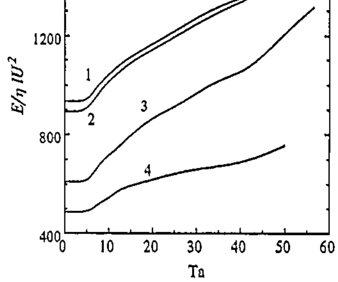

Fig. 2. Dependence of energy dissipation on the Taylor number Ta: $\delta = 0.2$; $R = 10$ (1, 2, 3), 5 (4); $b = 2$ (1, 2, 4), 1 (3); $\alpha = 45°$, $R_k = 1.22$ (1, 3, 4) $\alpha = 30°$, $R_k = 1.05$ (2).

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