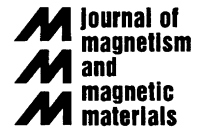




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Kelvin force in a layer of magnetic fluid

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Abstract

The Kelvin force in a layer of magnetic fluid subjected to a homogeneous magnetic field and local heating is studied. The study is motivated by the question about the corresponding Kelvin force density (Phys. Rev. Lett. 84 (2000) 2762). It is shown that the usual and the newly proposed formulation of the Kelvin force are entirely equivalent. It is only when approximations are introduced that differences arise. © 2002 Elsevier Science B.V. All rights reserved.

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Two prominent directions of interest can be identified among the present studies of phenomena occurring in magnetic fluids (MFs). These are the study of unconventional fluid dynamics phenomena such as the “negative viscosity” effect and the Weissenberg effect (for a review see Ref. [1]) and the proposal of new theoretical concepts [2]. Both directions are interwoven in the observation of a novel convective instability in a horizontal MF layer [3,4] and in the subsequent discussion about the correct form of the magnetic (or Kelvin) force density [5,6].

This first discussion about the range of validity of the Kelvin force was followed by a second one [7,8] triggered by a paper announcing that a pendulum experiment had confirmed the invalidation of the Kelvin force in MFs [9]. The claim was not accomplished according to Ref. [7] and the resulting need for a clarification entailed an extended paper [10]. A clarification as in the

pendulum experiment is lacking for the convection experiment. The aim of the present paper is to show that the usual and the proposed formulation of the Kelvin force are entirely equivalent. It is only when one introduces approximations that differences arise.

In Refs. [3,4] a horizontal layer of MF (stable colloidal suspension of magnetite nanoparticles dispersed in kerosene) between two glass plates is locally heated by a focused laser beam. It passes perpendicularly through the layer in the presence of a homogeneous vertical magnetic field. The absorption of the light by the fluid generates a temperature gradient and subsequently a refractive index gradient. This gradient is optically equivalent to a diverging lens, leading to an enhancement of the beam divergence. As a result, depending on the strength of the magnetic field different diffraction patterns appear [4].

To explain the observed phenomena, the form of the magnetic force inside the fluid has to be known. Therefore, a horizontal layer of MF is considered which is subjected to a homogeneous vertical magnetic induction. Since the temperature

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and the concentration of the fluid may vary, the magnetic field in the fluid is inhomogeneous and give rise to a finite Kelvin force density, \mathbf{f}_K . It can be derived from the Helmholtz force [11]

$$\mathbf{f}_K = \mu_0 \text{grad} \left[\frac{H_{\text{int}}^2}{2} \rho \frac{\partial \chi}{\partial \rho} \right] - \mu_0 \frac{H_{\text{int}}^2}{2} \text{grad} \chi, \quad (1)$$

where $\chi = \alpha\rho(1 + \beta_1\alpha\rho)$ is the susceptibility of the MF, ρ its density, H_{int} the absolute value of the magnetic field inside the fluid, and $\mu_0 = 4\pi \times 10^{-7}$ H/m. Higher-order terms in ρ are included in χ in order to determine the Kelvin force beyond the dilute limit. This limit is given by $\beta_1 \equiv 0$, i.e. $\chi = \chi_L$ which is the susceptibility according to Langevins theory which assumes noninteracting particles. In this approximation χ_L depends linearly on the density, $\chi_L = \alpha\rho = \mu_0 m^2 \rho / (3kTm_{\text{eff}})$, where m_{eff} is the effective mass of a ferromagnetic particle with its ‘attached’ carrier liquid molecules [12], m the magnitude of the magnetic moment of the particles, T the temperature, and k the Boltzmann constant. The coefficient β_1 of the quadratic term in ρ was determined in different microscopic models [13–15], all of which provide the same value $\beta_1 = 1/3$. In the presence of a uniform external magnetic induction \mathbf{B}_{ext} , the internal field is given by $\mathbf{H}_{\text{int}} = \mathbf{B}_{\text{ext}} / (\mu_0(1 + \chi))$. Inserting all expressions in Eq. (1), the Kelvin force follows as

$$\mathbf{f}_K(\chi) = - \frac{B_{\text{ext}}^2 \chi_L^2 \{1 + \beta_1 [3\chi_L(1 + \beta_1\chi_L) - 1]\}}{\mu_0 (1 + \chi)^3} \times \frac{\text{grad} \chi_L}{\chi_L}. \quad (2)$$

In Ref. [5] a variant form for the Kelvin force is proposed. By defining a different susceptibility $\bar{\chi}$ via $\mathbf{M} = (\bar{\chi}/\mu_0)\mathbf{B}_{\text{int}}$ with $\bar{\chi} = \chi/(1 + \chi)$ the Helmholtz force has now the variant form [5]

$$\mathbf{f}_V = \text{grad} \left[\frac{B_{\text{int}}^2}{2\mu_0} \rho \frac{\partial \bar{\chi}}{\partial \rho} \right] - \frac{B_{\text{int}}^2}{2\mu_0} \text{grad} \bar{\chi} \\ = \frac{B_{\text{int}}^2}{2\mu_0} \text{grad} \left[\rho \frac{\partial \bar{\chi}}{\partial \rho} - \bar{\chi} \right] + \frac{\rho}{2\mu_0} \frac{\partial \bar{\chi}}{\partial \rho} \text{grad} B_{\text{int}}^2. \quad (3)$$

$\mathbf{B}_{\text{int}}(\mathbf{M})$ is the magnetic induction (magnetization) in the fluid and B_{int} its absolute value. Due to the uniform form of the external induction and the continuity of the magnetic induction across the

interface, the last term is zero and the first term gives with the definition of $\bar{\chi}$

$$\mathbf{f}_V(\chi) = \frac{B_{\text{ext}}^2}{2\mu_0} \text{grad} \left[\frac{\beta_1 \alpha^2 \rho^2 - \chi^2}{(1 + \chi)^2} \right]. \quad (4)$$

Executing the differentiation in Eq. (4) leads exactly to the same result as in Eq. (2). Therefore, both formulations are indeed physically equivalent under the inclusion of a quadratic term in ρ . The equivalence is true also for higher terms in ρ provided that the susceptibility can be written as $\chi = \alpha\rho[1 + \sum_{i=1}^{\infty} \beta_i(\alpha\rho)^i]$. Thus there is no a priori reason to prefer Eq. (3) over Eq. (1) because both formulations lead to the same result as long as the definition of $\bar{\chi}$ is used in Eq. (3). This very basic fact independent on the relation of χ on ρ has to be emphasized since it recedes in the wake of the discussion [5,6].

The discussion about the range of validity of \mathbf{f}_K in Ref. [5] is based on the *simultaneous* approximation that $\chi \sim \rho$ and $\bar{\chi} \sim \rho$. The concurrent correctness of both relations has to be checked very cautiously. Since $\rho = \chi_L/\alpha$ with constant α , the proportionality to the density ρ is equivalent with the proportionality to the Langevin susceptibility χ_L . Restricting the dependence of the susceptibilities on χ_L up to the third order, one has

$$\chi = \chi_L [1 + \beta_1 \chi_L + \beta_2 \chi_L^2] + O(\chi_L^4) \quad (5)$$

and

$$\bar{\chi} \simeq \chi(1 - \chi + \chi^2 - \dots) \\ = \chi_L [1 + (\beta_1 - 1)\chi_L + (\beta_2 + 1 - 2\beta_1)\chi_L^2] + O(\chi_L^4), \quad (6)$$

where expansion (6) is valid for $\chi \ll 1$ only. Assuming a linear dependence of χ on χ_L , i.e. $\beta_1 = \beta_2 = 0$, expansion (6) implies necessarily that $\bar{\chi}$ depends on higher-order terms of χ_L . Fig. 1 shows the *linear* behaviour of $\chi = \chi_L$ (dashed line) and the *nonlinear* behaviour of $\bar{\chi} = \chi_L(1 - \chi_L + \chi_L^2)$ (solid line) for $0 \leq \chi_L \leq 0.5$. A nonlinear dependence of $\bar{\chi}$ on χ_L in the region, where $\chi \sim \chi_L$ holds, is confirmed also by measurements (see Fig. 1(b) in Ref. [6]).

From Fig. 1 it becomes evident that in the region $\chi = \chi_L$ a subregion $\chi_L \leq 0.06$ exists, where additionally $\bar{\chi} = \chi_L$ is fulfilled. Inserting $\chi = \chi_L$ in

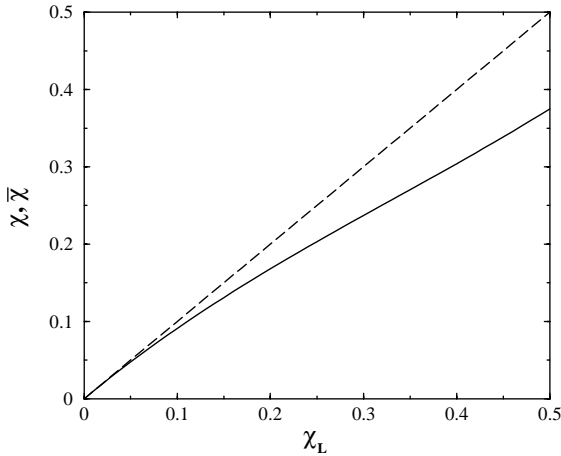


Fig. 1. Nonlinear dependence of the susceptibility $\bar{\chi} = \chi_L(1 - \chi_L + \chi_L^2)$ (solid line) on the Langevin susceptibility χ_L . The dashed line shows the linear function $\chi = \chi_L$.

Eqs. (2) and (4), the resulting force density

$$\mathbf{f}_K(\chi) = \mathbf{f}_V(\chi) = -\frac{B_{\text{ext}}^2 \chi_L \text{grad } \chi_L}{\mu_0 (1 + \chi_L)^3} \quad (7)$$

is nonzero. This agreement confirms the above general statement that the usual and the variant form of the Kelvin force density are equivalent provided they are functions of χ (see Eqs. (2) and (4)). But inserting $\chi = \chi_L$, $\mathbf{M} = \chi \mathbf{H}_{\text{int}}$ in Eq. (1) and $\bar{\chi} = \chi_L$, $\mathbf{M} = (\bar{\chi}/\mu_0) \mathbf{B}_{\text{int}}$ in Eq. (3), respectively, one gets

$$\mathbf{f}_K = \mu_0 (\mathbf{M} \text{grad}) \mathbf{H}_{\text{int}} \quad (8)$$

versus

$$\mathbf{f}_V = (\mathbf{M} \text{grad}) \mathbf{B}_{\text{int}}, \quad (9)$$

where the first expression gives a nonzero force density equal to (7), it is zero in the second case. The reason for the difference between the nonzero result of Eq. (7) and the zero one of Eq. (9) is the following: the variant form of the Helmholtz force (3) is a direct function of any approximation of $\bar{\chi}$ whereas the correct definition of $\bar{\chi}$ was incorporated into $\mathbf{f}_V(\chi)$ (see Eq. (4)). That is the deeper reason why approximations cause differences if the two formulae for the Helmholtz force are used. It has to be noted that this discrepancy is limited to a small subregion $\chi = \chi_L \leq 0.06$ which is outside the

usual experimental fluids. The lowest susceptibility of commercially available fluids is 0.13¹.

For $\chi = \chi_L > 0.06$ a truncation in the expansion of $\bar{\chi}$ after the linear order is deficient (see Fig. 1). If one inserts instead the entire term $\bar{\chi} = \chi_L(1 - \chi_L + \chi_L^2)$ in Eq. (3), one obtains a nonzero force density also for \mathbf{f}_V ,

$$\mathbf{f}_V = -\frac{B_{\text{ext}}^2}{\mu_0} (1 - 3\chi_L) \chi_L \text{grad } \chi_L, \quad (10)$$

which is a good approximation of Eq. (7) for small χ_L .

These theoretical calculations as well as the experimental measurements in Ref. [6] show apparently that (i) a linear dependence of χ on the density results not necessarily in a linear dependence of $\bar{\chi}$ on the density, and (ii) nonlinear contributions of ρ are relevant for $\bar{\chi}$ even in the region $\chi \ll 1$. (iii) The two formulae for the Helmholtz force are entirely equivalent. It is only when one introduces approximations that differences arise in a small subregion, $\chi = \bar{\chi} = \chi_L \leq 0.06$.

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¹Data sheets of Ferrofluidics Corporation.