ELSEVIER
Journal of Magnetism and Magnetic Materials 252 (2002) 271–275
Journal of magnetism and magnetic materials
www.elsevier.com/locate/jmmm

The dynamic behavior of a collapsing bubble in a magnetic fluid

F.R. Cunha[a,*], A.J. Sousa[a], P.C. Morais[b]

[a] *Departmento de Engenharia Mecânica, Universidade de Brasília, 70919-970, Brasília-DF, Brazil*
[b] *Instituto de Física, Núcleo de Física Apicada, Universidade de Brasília, 70919-970, Brasília-DF, Brazil*

Abstract

In this article it is investigated the nonlinear response of an oscillatory bubble suspended in an incompressible magnetic fluid. After an appropriate non-dimensionalization of the governing equation, it is found that the most relevant physical parameters of the system are: the Reynolds number, the Weber number, the magnetic pressure coefficient and the magnetic permeability ratio bubble-fluid governing equation. The integration of the nonlinear differential equation governing the bubble motion is performed analytically by using a regular expansion and numerically by using a fourth-order Runge–Kutta scheme. Unstable configurations of the bubble motion are shown for different values of the magnetic pressure coefficient. An important consequence of magnetic colloidal particles in the flow is that it may drastically attenuate instabilities, avoiding bubble against collapse. The present findings have implications for acoustic cavitation in cryogenic liquids.

Keywords: Bubble; Magnetic fluid; Collapse; Oscillatory motion

1. Introduction

In transient cavitation the transformation of low-energy density sound wave into a high-energy density collapsing bubble occurs due to the motion is nonlinear. A collapsing bubble concentrating energy into very small volumes may produce very high pressures and temperatures which can erode solids like vanes of a turbine in a hydroelectric plant and produce sonoluminescence [1,2]. The cooling process of high-power electric transformer by natural circulation of a polarized fluid under the action of magnetic field is a related problem. A first study of the phenomenon of bubble collapse in fluid was proposed by Rayleigh [3]. In the limit where an imposed acoustic field has a small Mach number and the wavelength of the sound field is large compared to the bubble radius, one is led to the leading order nonlinear Rayleigh–Plesset equation [4–7]. Acoustic cavitation has provided a strong incentive for the study of the dynamics of gas bubbles in oscillating pressure field, but the understanding of the corresponding problem for gas bubbles in a magnetic fluid is, relatively speaking, less developed. The purpose of this work is to extend the Rayleigh–Plesset model to accounting for the magnetic effects on the nonlinear response of an oscillating bubble in a magnetic fluid. The model accounts in detail for the magnetic-fluid-mechanic processes in the bubble interface.

2. Continuum formulation

The complete problem of a gas bubble undergoing nonlinear radial pulsations in a ferrofluid is a complex problem, as its solution requires a consideration of the equations of conservation of mass, momentum in the magnetic liquid and in the gas, magnetostatic limit of Maxwell's equations coupled by suitable interface conditions. The problem falls naturally into two

*Corresponding author.
E-mail address: frcunha@unb.br (F.R. Cunha).

0304-8853/02/$ - see front matter © 2002 Elsevier Science B.V. All rights reserved.
PII: S 0 3 0 4 - 8 8 5 3 (0 2) 0 0 6 2 2 - 4

parts: that of finding the magnetic field and that of determining the fluid motion with appropriated boundary conditions on the bubble interface. It is considered a bubble of initial radius a oscillating in an incompressible continuum magnetic fluid of viscosity η and density ρ. The fluid within the bubble is considered to be a perfect gas describing a politropic process $p_b V^n = C$, where p_b is the internal absolute pressure of the bubble, V is the bubble volume at any time and n is the politropic coefficient. The magnetic fluid is subject to an acoustic perturbation like $p_\infty(t) = p_0(1 - \varepsilon \sin \omega t)$, where p_0 is the static pressure, ε the acoustic pressure amplitude and ω the sound angular frequency. An imposed uniform magnetic field $H_0\boldsymbol{k}$ with small intensity H_0 is also assumed so that the bubble remains slightly spherical and thus its surface $r = R(t)$ develops a pure radial motion (i.e. the bubble elongation along the field direction is smooth and the bubble shape has just a small disturbance in the surface curvature). The magnetic permeability of the gas μ_2 is much smaller than the permeability μ_1 of the magnetic fluid surrounding the bubble. In this limit $\mathbf{H}$ within the bubble is zero, and the field lines incident on the sphere are purely radial [8–10].

One considers the situation of flows with slowly shifting orientation of magnetic field relative to translating and rotating fluid particles. So, magnetization $\mathbf{M}$ is parallel to the magnetic field $\mathbf{H}$, and antisymmetric stresses and couples may be neglected. Magnetization $\mathbf{M}$ is defined in $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, where μ_0 is the magnetic permeability of free space and $\mathbf{B}$ is the magnetic induction. $\mathbf{H}$ and $\mathbf{B}$ satisfy the magnetostatic limit of Maxwell equations in the absence of electric currents, $\nabla \times \mathbf{H} = 0$ and $\nabla \cdot \mathbf{B} = 0$ [8]. $\mathbf{H}$ may be expressed in terms of a magnetic potential $\mathbf{H} = \nabla\psi$, satisfying $\nabla^2\psi = 0$.

The balance of mass and momentum for the fluid flow are given, respectively, by

$$\nabla \cdot \boldsymbol{u} = 0, \quad \rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g}, \tag{1}$$

where $\boldsymbol{\sigma}$ is the fluid stress tensor, $\boldsymbol{u}$ is the velocity field and $\boldsymbol{g}$ is the gravity acceleration. For a polarized fluid the magnetic contribution for the stress tensor takes into account a magnetic pressure and an extra stress in the direction of the magnetic field [8], namely

$$\boldsymbol{\sigma} = -(p + p^*)\mathbf{I} + 2\eta\mathbf{E} + \mu_0(1 + \chi)H^2 ss, \quad \mathbf{x} \in D_1. \tag{2}$$

Here p denotes the mechanical pressure, $p^* = (1/2)\mu_0 H^2(1 + 2\bar{\chi})$ is the magnetic pressure for an incompressible and isothermal media, $\mathbf{I}$ is the isotropic unity tensor, H is the magnitude of the field acting in direction of the unit vector $\mathbf{s}$, $\bar{\chi} = \bar{M}/H$ is the field averaged susceptibility based on the averaged magnetization, $\chi = M/H$ is the susceptibility, $\mathbf{E} = (\frac{1}{2})(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}})$ is the rate of strain tensor. Subscript T denotes transpose tensor and D_1 and D_2 the region occupied by ambient ferrofluid and bubble, respectively (see schematic in Fig 1). As the gas inside the bubble is considered inviscid and non-magnetic Eq. (2) reduces simply to $\boldsymbol{\sigma} = -p\mathbf{I}$ if $\mathbf{x} \in D_2$.

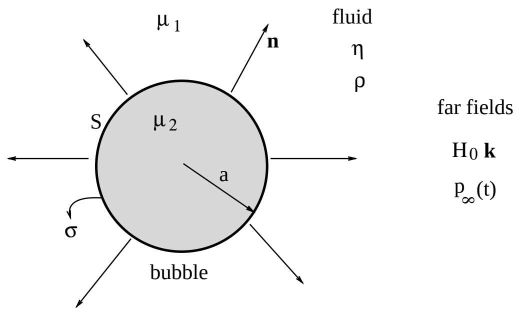

Fig. 1. Schematic representation of a bubble undergoing non-linear pulsations in a polarized fluid.

At lowest order elongation the bubble surface S may here be taken as $r = R(t)$. If the bubble permeability becomes small, the magnetic field within the drop becomes small. In particular, the tangential stress becomes small. Thus, the boundary conditions require a continuous velocity at the bubble interface $u_1 = u_2$ and a balance between the net normal surface traction and surface tension forces. For a clean interface the jump of normal traction calculated by using Eq. (2) is given by $\sigma_{nn} = (\boldsymbol{n} \cdot s \cdot \boldsymbol{n})_1 - (\boldsymbol{n} \cdot s \cdot \boldsymbol{n})_2 = 2\kappa\sigma$, where $\boldsymbol{n}$ is the unit normal vector to S and κ is the surface mean curvature. Also, the normal component of $\mathbf{B}$ and the tangential component of magnetic field $\mathbf{H}$ are continuous across the bubble interface.

Now making use of the balance and the constitutive Eqs. (1) and (2) and applying the boundary conditions on the bubble interface for a constant susceptibility magnetic fluid with $\mu_1 \gg \mu_2$, after an appropriate non-dimensionalization the dimensionless governing equation for describing the bubble interface motion have the form

$$R\ddot{R} + \left(\frac{4}{Re} + \frac{3}{2}R\dot{R}\right)\frac{\dot{R}}{R} = -F(t, R), \tag{3}$$

where the function $F(t, R)$ is given by

$$F(t, R) = 1 + \varepsilon \sin \omega t + \frac{2}{We}\frac{1}{R} - \left(1 + \frac{2}{We}\right)\frac{1}{R^{3n}} + C_{pm}\left(1 + \frac{2}{\chi}\right), \tag{4}$$

where $Re = \rho a U_c/\eta$ (i.e. ratio of inertia force to viscous force), $We = \rho a U_c^2/\sigma$ (i.e. ratio of inertia force to interfacial force) and $C_{pm} = (1/2)\mu_0\chi^2 H_0^2/(\rho U_c^2)$ (i.e. ratio of magnetic force to inertia force) are, respectively, the Reynolds number, Weber number and the magnetic pressure coefficient. These physical parameters are defined after a scaling analysis of the problem by considering a the characteristic length scale and $U_c = (p_0\rho^{-1})^{1/2}$ the characteristic velocity of the motion. The nonlinear Eq. (3) is numerically integrated for an initial dimensionless radius $R(0) = 1$ and an interfacial velocity $\dot{R}(0) = 0$.

3. Results and discussion

The nonlinear governing Eq. (4) is integrated by using a fourth-order Runge–Kutta scheme to find the bubble radius evolution and the pressure history of the gas inside the bubble. The bubble is forcing by a time-varying pressure as a sinusoidal standing wave. As a preliminary test, the dimensionless collapse time $\tau \sim 0.915$ predict by the exact solution of Rayleigh [1] was recovered by the simulation with a dimensionless time step of 10^{-3}. The error between exact and numerical solutions was smaller than 0.1%. We have also validated our calculations by repeating them many times with different values of the numerical parameters such as the time step, always with the same results at least up to several cycles. A regular asymptotic expansion for small amplitude of the pressure forcing has been also developed [11] to capture the first-order solution of the nonlinear governing Eq. (4).

A typical sequence of bifurcations in the nonlinear response of the bubble is shown in Figs. 2,3 and 4 for $\varepsilon = 1$, $Re = 100$, $We = 10$ and $C_{pm} = 0$, 0.4 and 1.0, respectively. Fig. 2 shows that in the absence of the magnetic field the bubble collapses after a relatively short time of oscillations. For a dimensionless time about 4, the bubble radius starts growing immediately. The bubble wall reaches a limit size ($\approx 3a$) at time about 32. The bubble radius then starts to decrease and the pressure increases drastically (see Fig. 2b) until the bubble collapses when it reaches its hard core. The opened phase diagram shown in Fig. 2c corresponds to the dynamic condition of collapse. Fig. 3 shows the same plots in the presence of a magnetic field for $C_{pm} = 0.4$. It can be seen a nonlinear response of the bubble without collapse. Comparing the behavior of the internal pressure of the bubble and bubble radius, it is observed that when the radius reaches a minimum the pressure takes the high amplitude. At this point the bubble interface cannot hold the pressure and it starts growing until the pressure reaches a minimum again. The period of oscillations may form closed deformed curves at the phase diagram (see Fig. 3) depending on the degree of nonlinearity in the bubble motion. A drastic change in the bubble nonlinear motion is shown in Fig. 4 for $C_{pm} = 1$. As can be seen from the phase diagram in Fig. 4, even a small effect of a magnetic field may dynamically stabilize a bubble motion from a configuration of collapse to one just weakly unstable. A key finding of the present investigation was that the oscillatory motion of a gas bubble in a ferrofluid induced by a sufficiently large acoustic pressure might

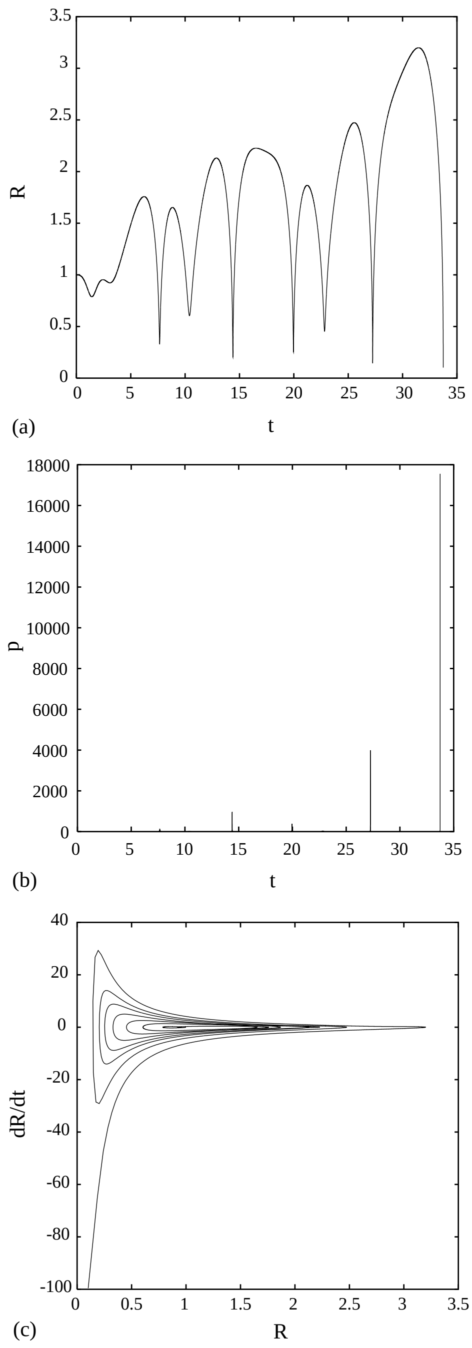

Fig. 2. Bubble motion for $Re = 100$, $We = 10$, $\varepsilon = 1$ and $C_{pm} = 0$: (a) Bubble radius as a function of time, (b) pressure inside the bubble and (c) phase diagram.

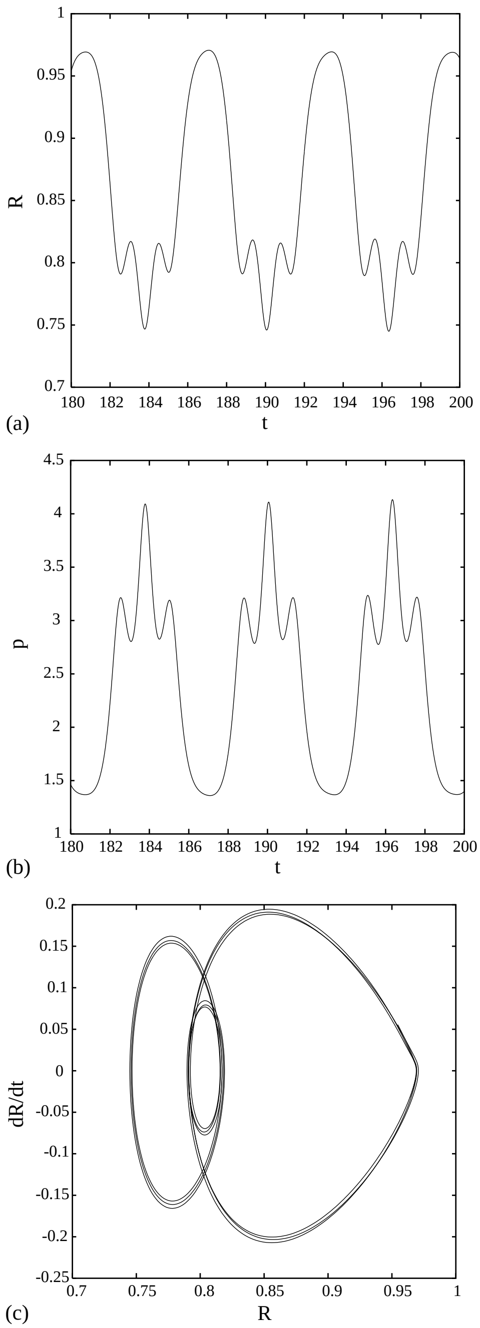

Fig. 3. Same caption of Fig. 2 for $C_{pm} = 0.4$.

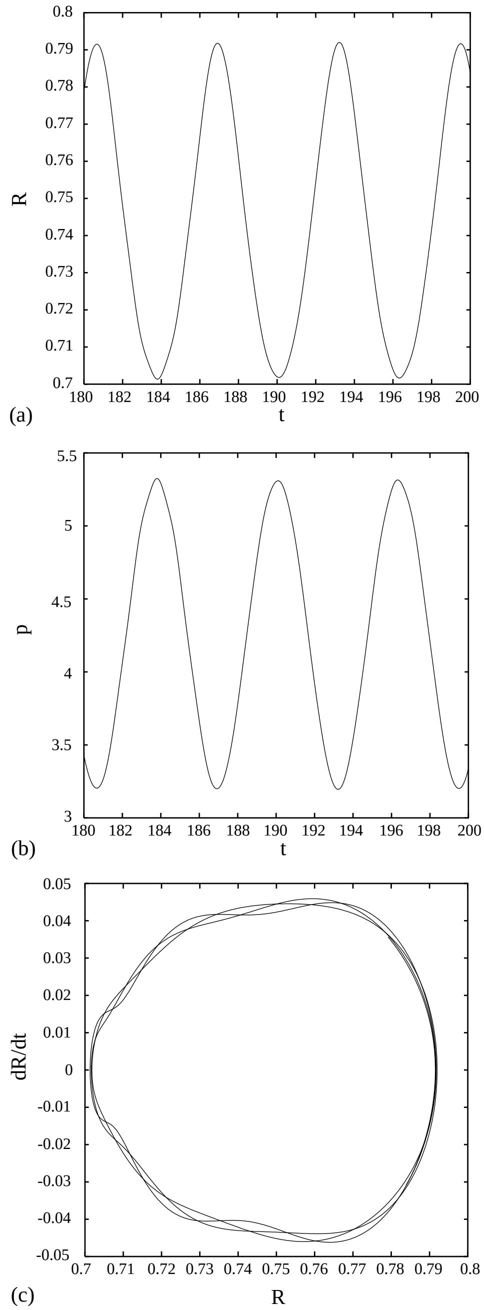

Fig. 4. Same caption of Fig. 2 for $C_{pm} = 1.0$.

be significantly attenuated without exhibiting a chaotic response when the magnitude of the magnetic pressure is comparable to the inertia force acting on the bubble interface. This magnetic effect is a direct consequence of an extra normal stress produced in the surrounding polarized fluid under the action of a field. This process could have implications for preventing bubbles against collapse and consequently cavitation in turbo-machines. Experimental results are required to obtain more definite answers.

Acknowledgements

This work was partially supported by the Brazilian agencies CNPq, CAPES and CT-Petro-Finep.

References

[1] F.R. Young, Cavitation, McGraw-Hill, New York, 1989.
[2] B.P. Barber, R.A. Hiller, R. Lofstedt, S.J. Putterman, K.R. Weninger, Phys. Rep. 281 (1997) 65.
[3] L. Rayleigh, Phil. Mag. 34 (1917) 94.
[4] M.S. Plesset, Trans. ASME. J. Appl. Mech. 71 (1949) 277.
[5] M.S. Plesset, T.P. Mitchell, Q. Appl. Math. 13 (1956) 419.
[6] B. Noltingk, E. Neppiras, Proc. R. Soc. B63 (1950) 674.
[7] A. Prosperetti, Rend. Sc. Int. Fis. XCIII (1984) 145.
[8] R.E. Rosensweig, J. Appl. Phys. 57 (1985) 4259.
[9] J.C. Bacri, D. Salin, J. Phys. Lett. 44 (1983) 415.
[10] V.I. Arkhipenko, Yu.D. Barkov, V.G. Bashtovoi, Magnetohydrodynamics 14 (1978) 373.
[11] Y. Hao, A. Prosperetti, Phys. Fluids 11 (1999) 2019.