

Convection driven by forced diffusion in magnetic fluids under the influence of strong magnetic field gradients

Stefan Odenbach

Institut für Materialwissenschaften, BUGH Wuppertal, Müngstener Str. 10, D-42285 Wuppertal, Germany

Abstract

We have investigated the convective flow generated by the interaction of a magnetic field gradient with a gradient in magnetization in a magnetic fluid. This gradient is caused by the diffusion of the magnetic particles in the field gradient.

1. Introduction

Because of their colloidal make-up and their long-term stability, which allows reproduction of experiments, ferrofluids are excellent model substances for many problems. This is, for example, valid for the high-gradient magnetic separation (HGMS) of magnetically marked particles from carrier liquids. This process, used e.g. for many medical problems like separation of blood cells or extraction of lymphocytes (see e.g. Refs. [1–3]), has been the subject of a large number of theoretical and practical investigations. Part of the work was focused on the final state of the demixing process of a suspension of magnetic particles in a carrier liquid subjected to a strong magnetic field gradient. Another part of the investigations dealt with a special effect taking place during the beginning of the demixing process. Shortly after a non-homogeneous magnetic field gradient is applied to the suspension, a gradient in concentration of the magnetic component antiparallel to the magnetic field gradient develops. The interaction between the gradient in magnetization in the suspension, which is given by the concentration gradient, and the field gradient itself gives rise to a convective flow [4]. This non-equilibrium process is of great importance for the demixing process since it amplifies the velocity of demixing over the normal diffusion velocity. Nevertheless, only indirect experimental verification of the phenomenon [5] had been given for many years due to measurement problems.

2. Experiments

In our experiments we have chosen a cylindrical geometry consisting of a magnetic fluid enclosed between two concentric cylinders under the influence of an azimuthal magnetic field produced by a current leading wire in the cylinder axis (see Fig. 1). The azimuthal field provides a

radial gradient forcing the demixing process. This geometry was chosen in comparison with theoretical investigations carried out by Chukhrov [4].

In a number of experiments carried out in such a cylindrical geometry we have measured the evolution of the radial concentration distribution between the concentric cylinders as a function of time. Small samples of the fluid were taken out of the container, and their concentration was determined by measuring the inductance of a small coil filled with the fluid. The change of inductance of the coil is proportional to the concentration of the magnetic particles in the fluid sample. With this method we were able to determine the relative concentration of magnetic particles (normalized to equilibrium concentration) to an accuracy of 2×10^{-4} .

We have also investigated the onset of convection by measuring the flow velocity by means of a thermal anemometer. The anemometer consists of two microthermistors with a mean diameter of 0.5 mm. The thermistors are placed in the fluid layer as shown in Fig. 2. One of them is heated by a current of 1 mA to a temperature that is about 1.5°C higher than that of the surrounding. A flow of the fluid cools the thermistor and therefore gives rise to a change of resistance of the probe. The second thermistor is used to monitor temperature changes of the whole

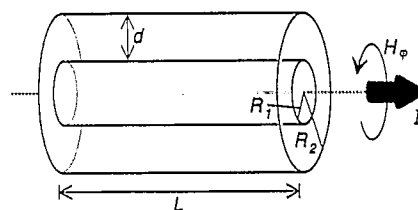


Fig. 1. Sketch of the experimental set-up showing the fluid layer enclosed between two concentric cylinders under the influence of an azimuthal magnetic field.

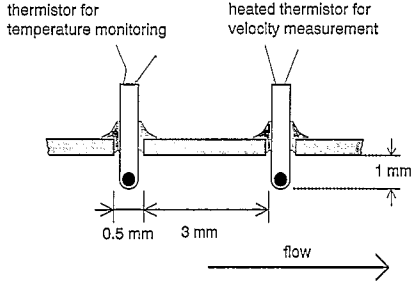


Fig. 2. The anemometer consists of two microthermistors. One is heated by a current and cooled due to the flow, while the second one monitors the fluids temperature.

system. This is necessary since the temperature change at the heated thermistor is only about 15 mK for a flow velocity of 0.25 mm/s. With the anemometer described above a velocity resolution of 0.03 mm/s was obtained.

3. Results

With the inductive method mentioned before we have measured the concentration distribution for different times after switching on the magnetic field. As an example Fig. 3 shows the distribution after a diffusion time of 10 h. In particular the increase of concentration antiparallel to the magnetic field gradient is clearly observed. The comparison with numerical calculations of the concentration distribution after 10 and 15 h shows that the diffusion develops 1.5 times faster than expected. This result is also obtained for diffusion times of 5, 7 and 15 h. It can be explained by the influence of the convective flow induced by the interaction between the concentration gradient and the magnetic field gradient. Blums et al. predicted [5] that such a diffusion should increase the diffusion velocity without changing the concentration distribution profile. Nevertheless it can not be excluded that the difference between the measurement and the calculation for the same diffusion time is given by some uncertainty in the knowledge of the

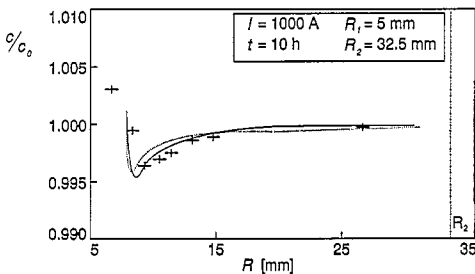


Fig. 3. The measured concentration values normalized to the equilibrium concentration c_0 after 10 h of diffusion time are indicated by crosses. The dotted line gives the numerical calculation integrated over the fluid sample for a diffusion time of 10 h while the full line is calculated for 15 h. The main parameters of the experiment are given in the inset.

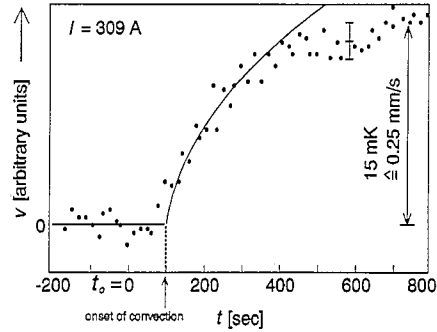


Fig. 4. The flow velocity of the convective flow as a function of time measured for an electric current $I = 309$ A producing the azimuthal magnetic field. The field is switched on at $t_0 = 0$.

particle size distribution since the size of the particles enters the numerical model in the sixth power.

With velocity measurements using the thermal anemometer described before, the existence of the convective motion was proven. Fig. 4 shows an example for the development of the velocity with time after the magnetic field is switched on at the time $t_0 = 0$. The maximum velocity of about 0.3 mm/s observed in the experiment is of the order of the predictions made by Blums [6]. The \sqrt{t} behaviour of the increase of velocity at the beginning of the convective process and the time delay between the beginning of the demixing — marked by switching on the magnetic field — and the onset of convection can be understood by using a time-dependent dimensionless parameter obtained from the Navier–Stokes equation. This magnetodiffusive Rayleigh number is given by

$$R_{md(t)} = \frac{\mu_0 \Delta M \nabla H_\phi d^3}{D_1 \eta}, \tag{1}$$

where ΔM denotes the magnetization differences in the fluid, ∇H_ϕ the gradient of the azimuthal magnetic field, d the thickness of the fluid layer, D_1 the diffusion coefficient of the particles in the fluid and η the fluids dynamic viscosity (μ_0 is the vacuum permeability).

Including the expressions for ∇H_ϕ and D_1 and using the time-dependent expression for the magnetization that can be obtained from the equation of diffusion, it can be written in the form

$$R_{md(t)} = \mathcal{E} \frac{I^4}{T^3 \eta} t. \tag{2}$$

where \mathcal{E} is a constant containing all fixed parameters of the experimental set-up, I is the current producing the azimuthal magnetic field and T is the temperature of the fluid. The temperature dependence of $R_{md}(t)$ is given by the temperature dependence of the magnetization, while the time dependence results from the fact that a non-stationary situation is considered. That means that the concentration distribution and therefore the magnetization and its

variation over the fluid varies with time due to the action of diffusion.

The convective motion starts when R_{md} exceeds a certain critical value R_{md}^* . If all parameters are fixed, it is obvious from Eq. (1) that some time delay between the beginning of demixing and the onset of convection must occur since R_{md} must increase from 0 at $t=0$ (start of demixing) to R_{md}^* at $t=t^*$ (start of convection). In addition the well known relation [7] between the amplitude of convection, that means the convective velocity and the reduced dimensionless parameter.

$$\alpha \sim \sqrt{\frac{R_{\text{md}} - R_{\text{md}}^*}{R_{\text{md}}}} \sim \sqrt{t - t^*} \quad (3)$$

yields directly the \sqrt{t} behaviour of the increase of velocity observed in the experiment shortly after the onset of convection.

The author thanks Professor Dr. K. Stierstadt (LMU München) for his support and many helpful discussions during the preparation of this work.

References

- [1] D. Melville, F. Paul and S. Roath, *IEEE Trans. Magn.* 18 (1982) 1680.
- [2] M.E. Lauva, Y.A. Plyavin'sh and L.S. Glinkina, *Magneto-hydrodynamics* 26 (1990) 457.
- [3] S. Roath, *J. Magn. Magn. Mater.* 122 (1993) 329.
- [4] A.Y. Chukhrov, *Magneto-hydrodynamics* 22 (1986) 254.
- [5] E. Blums and A.Yu. Chukhrov, *J. Magn. Magn. Mater.* 85 (1990) 210.
- [6] E. Blums, *J. Magn. Magn. Mater.* 65 (1987) 343.
- [7] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Oxford University Press, London, 1961).