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Taylor vortex flow of magnetic fluids under the influence of an azimuthal magnetic field

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Abstract

We have investigated the influence of an azimuthal magnetic field on the stability of Taylor–Couette flow. The stabilization of Couette flow due to the action of the magnetic field has been used to determine the rotational viscosity of the magnetic fluid as a function of the magnetic field strength. The results of the experiments are compared with theoretical calculations on rotational viscosity [1] and on Taylor–Couette flow in magnetic fluids [2]. If the size distribution of the magnetic particles is taken into account, good qualitative agreement is found. Any remaining quantitative discrepancies can be attributed to the interaction between the particles which is neglected in the theories.

1. Introduction

The flow of a fluid between concentric cylinders with the inner cylinder rotating has been of great interest for theoretical and experimental work since Taylor [3] published the first investigations showing a transition between laminar Couette flow and a vortex flow consisting of counter-rotating vortices, which today are called Taylor vortex flow. In the past 70 years many interesting topics have been considered by changing the geometry and the liquids used for the investigations. For example, the flow between eccentrically rotating cylinders [4] and the stability of the flow of liquid helium in a Taylor–Couette system [5] have been examined.

In recent years theoretical calculations and experiments have been performed concerning the behaviour of magnetic fluids in Taylor–Couette flow. These magnetic fluids are suspensions of small magnetic particles in appropriate carrier liquids. The particles (magnetite is most often used) have a mean diameter of 10 nm. Particles of this size contain only one magnetic domain. They are covered with surfactant to prevent agglomeration due to van der Waals attraction. In this way stable suspensions of the particles in carrier liquids such as water or oil – so-called ferrofluids – can be obtained. These suspensions show liquid behaviour coupled with superparamagnetic properties. This makes it possible to influence and control the flow and the properties of magnetic fluids with moderate magnetic fields [6,7]. This feature of magnetic fluids gives rise to a number of interesting hydrodynamic effects such as the existence of thermal convection in closed containers under microgravity [8,9], thermal convection in a fluid layer heated from above [10], or the appearance of non-Newtonian properties depending on the strength of the applied magnetic field [11].

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The first theoretical investigations on Taylor–Couette flow in magnetic fluids were carried out independently by Niklas [2,12] and by Vislovich et al. [13]. In addition to strong variations in the flow profile as a function of the direction and the strength of the applied magnetic field [12], a stabilization of Taylor–Couette flow should occur. This stabilization is based on an anisotropic increase in the viscosity of the fluid forced by the action of the magnetic field. This increase in viscosity can easily be understood if one assumes that the magnetic particles contain only one magnetic domain, so that they can be treated as macroscopic magnetic dipoles. If they are under the influence of a magnetic field and shear flow, two torques are acting on the particles. The shear flow exerts a torque that leads to a rotation of the particles. If the axis of rotation is not parallel to the magnetic field, the magnetic moment of the particle is tilted against the field direction. In this case the magnetic field exerts a magnetic torque on the particle to realign the magnetic moment with the field direction. Because of this, the free rotation of the particles is hindered, resulting in an increase in the viscosity of the fluid. In the opposite case, when the axis of rotation is parallel to the magnetic field, the magnetic moment will not be tilted against the field direction. Therefore no repulsive magnetic torque will occur and the free rotation of the particles will not be hindered. Thus no increase in viscosity will be observed in this situation.

The resulting increase $\eta_r(H) = \eta_{tot}(H) - \eta_0$ in the total viscosity η_{tot} over the zero field value η_0 was calculated by Shliomis in 1972 [1]. He obtained η_r as a function of the strength of the magnetic field H and its direction relative to the vorticity of the flow $\boldsymbol{\Omega}$ expressed by the angle β between $\boldsymbol{\Omega}$ and $\boldsymbol{H}$ in the form

$$\eta_r = \frac{3}{2}\Phi'\eta_0 \sin^2\beta \frac{\alpha - \tanh\alpha}{\alpha + \tanh\alpha}, \tag{1}$$

where Φ' denotes the volume fraction of the particles, including their surfactant layers, and $\alpha = \mu_0 V M_d H_{ex}/kT$ (μ_0 is the vacuum permeability, H_{ex} is the external magnetic field, V is the volume of a particle without surfactant, and M_d is its magnetization).

The change in viscosity forces a stabilization of Couette flow. Therefore it is possible to use the transition from Couette flow to Taylor vortex flow to investigate the rotational viscosity of a ferrofluid. The situation of a fluid between concentric cylinders with the inner one rotating is characterized by the Reynolds number,

$$\mathrm{Re} = \frac{\omega R_1 d \rho}{\eta_{tot}}, \tag{2}$$

where d is the width of the gap between the cylinders, ω is the angular velocity, R_1 is the radius of the inner cylinder, and ρ denotes the density of the fluid. Laminar Couette flow becomes unstable at a critical value Re^*, depending on the radius ratio, but not on the fluid properties or the magnetic field strength. Since η_{tot} depends on the magnetic field and Re^* is constant, the critical angular velocity ω^* at which the transition from Couette to Taylor vortex flow occurs must also depend on the magnetic field strength

$$\omega^*(H) = \eta_{tot}(H)\frac{\mathrm{Re}^*}{R_1 d\rho}. \tag{3}$$

By comparing the critical angular velocities measured under the influence of a magnetic field with the zero field value, one can determine the rotational viscosity from

$$\frac{\eta_{tot}(H)}{\eta_0} = \frac{\eta_r(H) + \eta_0}{\eta_0} = \frac{\omega^*(H)}{\omega^*(0)}. \tag{4}$$

In discussions of the rotational viscosity of magnetic fluids it has become common to define a normalized rotational viscosity $S := \eta_r/\eta_0$ and a reduced rotational viscosity S^* which is independent of the properties of the flow:

$$S^* := \frac{S}{\sin^2\beta} = \frac{\eta_r}{\eta_0 \sin^2\beta} = \frac{1}{\sin^2\beta}\left(\frac{\omega^*(H)}{\omega^*(0)} - 1\right). \tag{5}$$

To evaluate S^* from the measured critical angular velocities the average value of $\sin^2\beta$ must be known. We calculated this value from numerical data on the velocity profile kindly provided by Lücke and Roth [14] to $\langle \sin^2\beta \rangle = 0.69$ for an azimuthal magnetic field [15].

The first experimental results of the determination of rotational viscosity in axial and radial magnetic fields using the stabilization of Couette flow were obtained by Holderied et al. [16] and Ambacher et al. [15]. The scope of our experiments was to use the stabilization of Couette flow in an azimuthal magnetic field to determine the rotational viscosity of a ferrofluid in this field geometry.

2. Experimental setup

For these measurements we developed the experimental setup shown schematically in Fig. 1. It consists of a hollow cylindrical container filled with ferrofluid. The rotating inner cylinder is immersed in

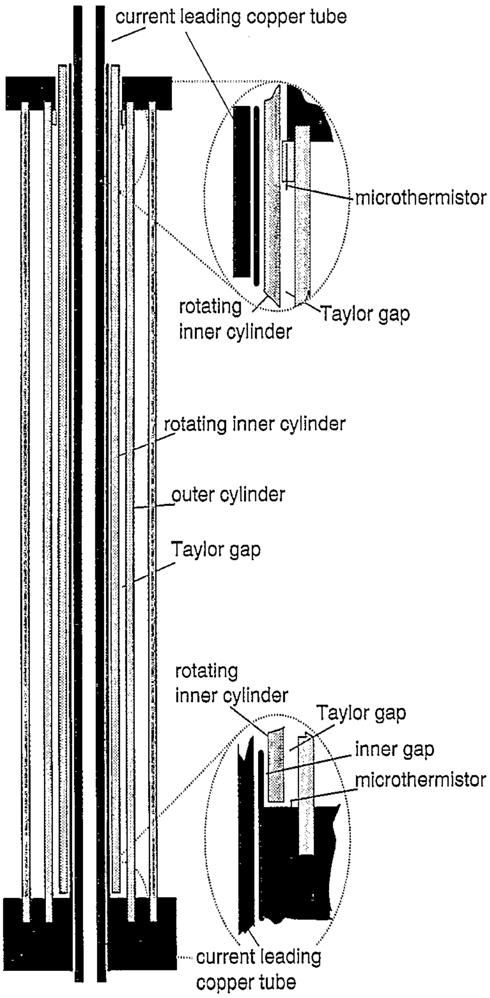

Fig. 1. The Taylor cell used for the investigations of rotational viscosity in ferrofluids. The features of the cell are explained in the text, and its specifications are given in Table 1.

Table 1
Parameters of the experimental setup

Radius of inner cylinder	R_1	16.00 ± 0.01 mm
Radius of outer cylinder	R_2	20.00 ± 0.01 mm
Gap width	d	4.00 ± 0.02 mm
Radius ratio	$\gamma = R_1/R_2$	0.8
Aspect ratio	$\Gamma = L/d$	56
Critical Reynolds number	Re^*	93.5 ± 0.4
Maximum current along copper tube	I_{max}	1400 A
Magnetic field strength in the centre of the outer fluid gap	H	122 ± 2 A/cm

the fluid and supported by ball bearings attached to the inner wall of the container. This geometry was chosen to allow the use of a copper tube along the cylinder axis leading an alternating current for the production of the azimuthal magnetic field without sealing problems. Due to this choice of geometry there are two fluid gaps in the system, connected by a narrow slit (0.5 mm thick) at the bottom of the container. The inner gap between the rotating cylinder and the inner wall of the container, shows only normal Couette flow, as in a system with a rotating outer cylinder. Therefore it does not contribute to the effects investigated here. In the outer gap between the rotating cylinder and the outer wall of the container Taylor vortex flow will occur for frequencies above the critical one. The cylinders were cooled with a water bath around the container to control the fluid temperature. The ferrofluid APG 513 A (Ferrofluidics Inc.) was used as the magnetic fluid. The relevant parameters of the experimental setup are given in Table 1, and the properties of the fluid are shown in Table 2.

The copper tube leading the electric current for the production of the azimuthal magnetic field was cooled with water. Nevertheless, some heating of the

Table 2
Properties of the magnetic fluid

Concentration of magnetic particles	Φ	6.7%
Mean particle diameter		10 nm
Kinematic viscosity (30°C)	υ	80.3 mm^2/s
Density	ρ	1.28×10^3 kg/m^3
Saturation magnetization	M_s	3.0×10^4 A/m
Magnetization of the magnetic particles	M_d	4.5×10^5 A/m

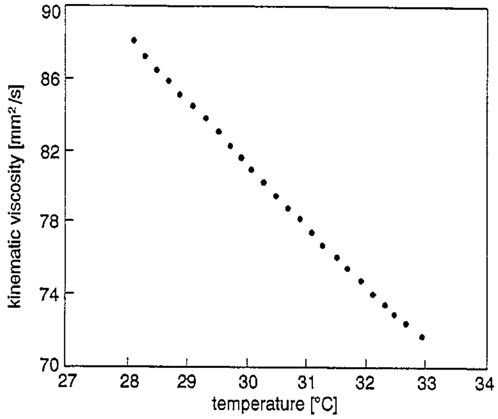

Fig. 2. Viscosity of the ferrofluid APG 513 A as a function of temperature. Further properties of the ferrofluid are given in Table 2.

ferrofluid due to the ohmic heat produced by the electric current was unavoidable. Since the viscosity of the fluid depends strongly on temperature (see Fig. 2), measurements of thermal variations were necessary in order to make the measured critical frequencies for different magnetic field strengths comparable. The temperature variations in the fluid were measured by means of two microthermistors placed at the top and the bottom of the fluid column (see Fig. 1). The thermistors were embedded in small glass tubes with mean diameters of about 0.5 mm. They have a resistance of 2 kΩ at 20°C with a temperature sensitivity of 100 Ω/K. Thus it is possible to measure temperature variations down to 0.01 K. The temperature homogeneity of the fluid in the column was checked by the use of heat-sensitive liquid crystals suspended in a silicon oil with a viscosity equal to that of the ferrofluid [17].

The critical angular velocity for the onset of Taylor vortex flow was determined from the torque necessary to drive the inner cylinder as a function of angular velocity. In both the Couette regime and the regime of Taylor vortex flow a linear relation holds for the dependence of the torque on the angular velocity. The difference between the two regimes is characterized by a difference in the slopes of the curves caused by the additional dissipation of rotational energy by the toroidal flow mode of the vortex flow. Therefore the critical angular velocity can be determined from the kink that occurs in the torque versus angular velocity curve (see Fig. 3).

3. Results and discussion

With the experimental setup described above we measured the critical angular velocity for 10 different azimuthal magnetic field strengths between 0 and 12.2 kA/m. Each of these measurements was repeated five times and the rotational viscosity was determined from the mean value of these data. The fluid temperature was fixed at 30°C with the water bath described in Section 2. All measured critical angular velocities were corrected to this common temperature using the temperature data obtained with

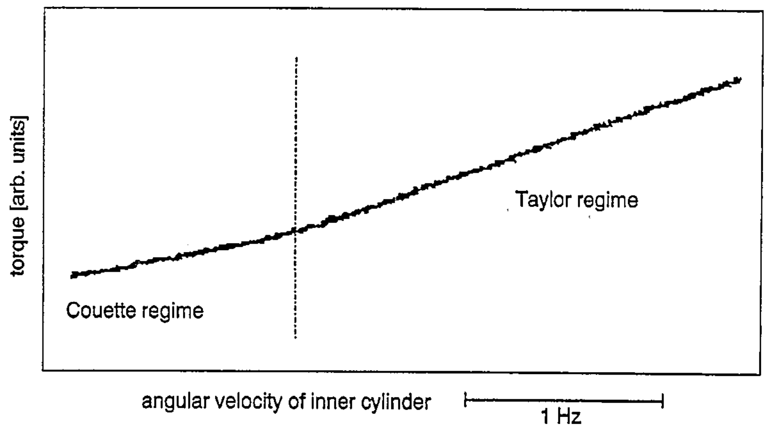

Fig. 3. The torque necessary for the driving of the inner cylinder as a function of angular velocity. The kink in the curve marks the onset of Taylor vortex flow.

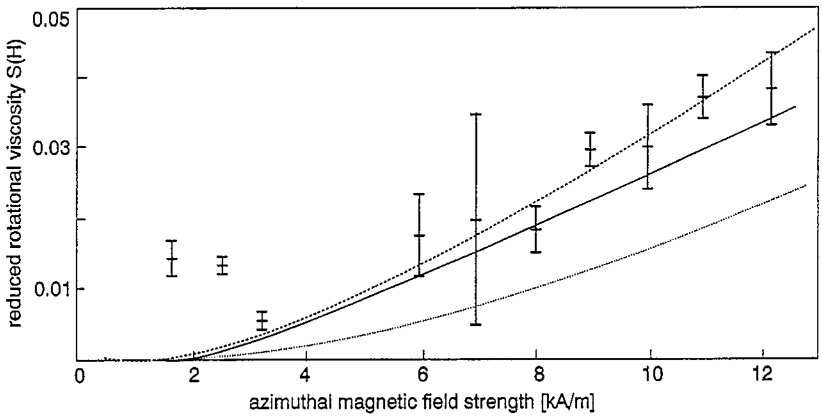

Fig. 4. Dependence of the normalized rotational viscosity S on the strength of the azimuthal magnetic field. The dotted line indicates the theoretical dependence calculated by Shliomis using the fluid data from Table 2 with a mean particle diameter of 10 nm. The full line uses the size distribution for the magnetic particles shown in Fig. 5. The dashed line represents the best fit to the experimental data.

the microthermistors. The reduced rotational viscosity was calculated from these corrected values for the angular velocity using Eqs. (4) and (5).

The results for S^* are shown in Fig. 4 as a function of the strength of the azimuthal magnetic field. The dotted line shows the theoretical behaviour of the reduced rotational viscosity as calculated from the Shliomis theory using the fluid properties given in Table 2 and a mean particle diameter of about 10 nm. It is clear that there is a remarkable discrepancy between the experimental results and the theoretical calculations. This discrepancy can be reduced if one takes into account that the fluid particles have a size distribution with particle sizes between 2 and 20 nm. The particle size distribution (see Fig. 5) for the fluid used in our experiments was obtained from a magnetization curve using a technique described in Ref.

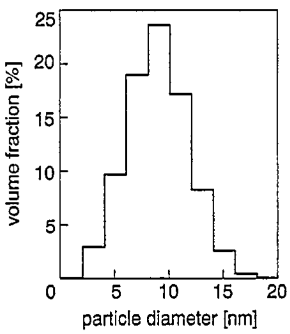

Fig. 5. Size distribution of magnetic particles obtained from the magnetization measurements.

[18]. The calculated reduced rotational viscosity using this size distribution is indicated by the full line in Fig. 4. The dashed line in Fig. 4 shows the best fit to the experimental data using a least-squares fitting technique. The remaining difference between this fit and the theoretical curve can be assumed to be due to the fact that the ferrofluid used for the experiments was a concentrated one (the volume fraction of the magnetic particles was around 10%), while the theoretical approach of Shliomis assumes a highly diluted suspension, which means that the interaction between the magnetic particles is neglected. Nevertheless this interaction is present in the concentrated suspensions used in the experiments. Our assumption that the remaining difference between the theoretical and experimental results is due to this neglect of interactions and not on mistakes made in the determination of the size distribution can be confirmed by comparing the experimental and theoretical values for the dependence of the ratio $\omega_c^2(S)/\omega_c^2(S=0)$ on the rotational viscosity. This dependence excludes all effects based on the particle size distribution since the size distribution enters the $S(H)$ and $\omega_c(S)$ in the same way. Fig. 6 shows that even in this dependence there is a discrepancy between our experimental results and the theoretical curve derived by Niklas [2].

Our results on the rotational viscosity of magnetic fluids obtained by investigating the onset of Taylor vortex flow under the influence of an azimuthal magnetic field, confirm previous investigations for

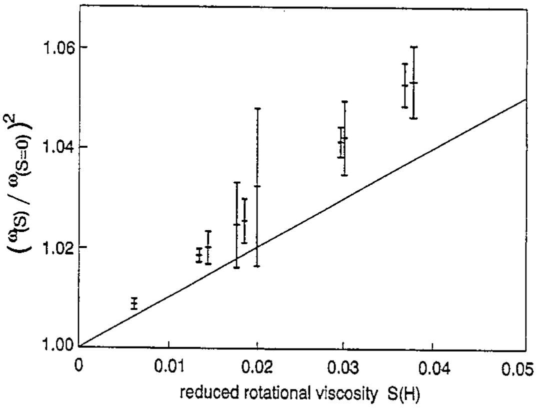

Fig. 6. Dependence of the ratio $\omega_c^2(S)/\omega_c^2(S=0)$ on the reduced rotational viscosity S^*.

axial and radial magnetic fields. In particular, it was found that a discrepancy between the theoretical calculations of the rotational viscosity and the experimental results remains even if the size distribution of the magnetic particles is taken into account. To rule out this discrepancy it would be necessary either to make a theoretical approach that includes the interaction between the magnetic particles, or to measure the rotational viscosity of highly diluted suspensions of magnetic particles.

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