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Continuous wave methods applied in determining the velocity of fast ultrasonic waves in saturated porous media

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Abstract. The application of measuring methods with continuous waves for study of the propagation velocity of fast ultrasonic waves in a liquid-saturated porous material is considered. The technique has recourse to (i) detection of the amplitude of interfered waves and (ii) detection of the phase of the transmitted wave, while the frequency varies linearly with time and the thickness of porous samples is constant. The two procedures permit convenient determination of the presence of dispersion and the velocity of the dispersionless waves. Relatively large amplitudes of excited continuous waves and high degree of accuracy of the methods recommend them as useful tools for study of dynamic properties of saturated porous materials.

1. Introduction

Dynamical measurements, in particular measurements of ultrasonic wave velocity in saturated porous materials, are of great practical importance in geophysics, biomechanics and materials engineering.

This paper concentrates on measurements in elastic permeable media saturated with a viscous liquid, that is in materials to which (at not excessively high frequencies) Biot's linear theory is well applicable [1].

With regard to their specific properties such materials cannot be dealt with, when it comes to choosing measuring techniques and interpretation of results, according to principles well established for single-phase materials.

Saturated porous media, as predicted by Biot's theory and confirmed experimentally, present the following fundamental properties.

(i) They admit propagation of two longitudinal waves [1, 4].
(ii) They are characterized by strong phase velocity dispersion in some (intermediate) frequency range [2, 3].
(iii) They exhibit considerable, strong frequency-dependent attenuation [2].

These properties originate in the two-phase nature of the medium and are also due to important interactions between the matrix and the liquid.

Although two longitudinal waves can propagate in the medium, one (the so-called slow wave) is so strongly attenuated that measurements of the velocity of the other (the so-called fast wave) are of greater practical significance.

The most commonly applied dynamical method used in studies of porous materials is the ultrasonic pulse technique [2, 4, 5].

In some frequency range the phase velocity and attenuation of the waves in porous materials are functions of the frequency. Thus, exact measurements by the pulse technique require use of pulses with a spectrum effectively excluding their deformation, or application of spectral analysis of the pulses entering the specimen and emerging from it [2, 6]. In the former case, one has to have available sufficient (not always accessible) knowledge concerning the material, as well as technical facilities enabling appropriate formation of pulses. In the latter case, construction of the measuring and analysing set-up involves considerable complications.

In the present investigation, application of the tunable frequency continuous wave method to measurement of the propagation velocity of longitudinal waves in a porous medium saturated with a liquid is considered.

The paper addresses the following two variations of the methods

(i) one making use of interference effects between the transmitted and reflected waves with amplitude detection of standing waves in the specimen,
(ii) another method involving detection of shift in

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phase between the wave transmitted by the medium and the wave emerging from a reference path.

The possibilities of the two techniques with regard to properties of porous materials, in particular, their velocity dispersion and strong attenuation, are discussed.

Continuous wave methods are classical laboratory techniques. Essentially, they are used when measuring propagation velocity of ultrasonic waves in liquids with interferometers with a variable measuring path [7].

In the configuration involving frequency tuning at a constant measuring length of the specimen, these methods are also suitable for measurements of the properties of solids and are then usually referred to as resonance methods [8].

The experimental set-up for measurement of phase velocities by the continuous wave method with tunable frequency, at a constant specimen thickness, are applied.

In order to exclude effects related to the slow wave, a configuration with transducers contiguous to the saturated specimen is used. In fact, the results of [9] show that the absence of relative motion at the frontal surfaces of the specimen ensures absence of the slow wave.

By judicious choice of the transverse dimensions of the specimens, any influence of their geometry on the results (effects of waves reflected from lateral surfaces) is practically eliminated.

2. Elementary description of propagation of acoustic waves in saturated porous media

Propagation of waves with low amplitude and not excessively high frequency in porous materials saturated with a liquid is described by the linear theory of Biot [1].

The one-dimensional equation of motion of the medium, on making use of the physical relationships for stresses in the matrix and the liquid as well as forces of interaction between the two phases, takes the form [2]

$$P\frac{\partial^2 u}{\partial x^2}+Q\frac{\partial^2 U}{\partial x^2}=\rho_{11}\frac{\partial^2 u}{\partial t^2}+\rho_{12}\frac{\partial^2 U}{\partial t^2}+b\frac{\partial}{\partial t}(u-U)$$

$$Q\frac{\partial^2 u}{\partial x^2}+R\frac{\partial^2 U}{\partial x^2}=\rho_{12}\frac{\partial^2 u}{\partial t^2}+\rho_{22}\frac{\partial^2 U}{\partial t^2}-b\frac{\partial}{\partial t}(u-U) \quad (1)$$

where u and U are, respectively, the displacements of the matrix and liquid; P, Q and R are the elastic constants of the medium; b is the flow resistance coefficient; ρ_{11}, ρ_{12} and ρ_{22}, are mass coefficients, related as follows to the porosity Φ and densities of the matrix ρ_s and liquid ρ_f [1]:

$$\rho_{11}+\rho_{12}=\rho_s(1-\Phi)$$

$$\rho_{22}+\rho_{12}=\rho_f\Phi.$$

The existence condition of non-trivial solutions of equation (1) in the form of longitudinal harmonic waves for displacements of the liquid leads to the following equation of dispersion [1, 2]:

$$Ak^4-(B\omega^2+\mathrm{i}E\omega)k^2+(C\omega^4-\mathrm{i}D\omega^3)=0 \quad (2)$$

where

$$A=PR-Q^2 \qquad B=P\rho_{22}+R\rho_{11}-2Q\rho_{12}$$

$$C=\rho_{11}\rho_{22}-\rho_{12}^2 \qquad D=b(\rho_{11}+2\rho_{12}+\rho_{22})$$

$$E=b(P+R+2Q)$$

where k is the complex wavenumber, and ω the angular frequency. The solution of the dispersion equation (2) in k gives four complex roots, two of which with positive real parts are physically meaningful and are related to propagation of two longitudinal waves in the saturated porous medium, namely a wave of the first, fast kind, and a wave of the second, slow kind [1, 3]. Theoretical considerations [1, 3] as well as experiment [2, 4] point to very strong attenuation of the slow wave. Hence, for practical purposes, description of the fast wave is of greater significance. Its phase velocity is [2]

$$v=\frac{\omega}{\mathrm{Re}(k)}=\left(\frac{2A}{B-[\frac{1}{2}(\Delta^2+B^2-4AC-E^2/\omega^2)]^{1/2}}\right)^{1/2} \quad (3)$$

with

$$\Delta^4=\left(B^2-4AC-\frac{E^2}{\omega^2}\right)^2+\frac{4}{\omega^2}(BE-2AD)^2.$$

The phase velocity determined from (3) is frequency-dependent. However, big variations in the velocity as a function of frequency occur only in the range of intermediate frequencies. In many natural materials, this is the 10^4–10^5 Hz range.

Throughout the region of low and high frequencies, the velocities of the fast wave vary insignificantly and can be approximated by the formulae [2]

$$v_0^2=\frac{P+R+2Q}{\rho_{11}+2\rho_{12}+\rho_{22}}$$

$$v_\infty^2=\frac{2A}{B-(B^2-4AC)^{1/2}}. \quad (4)$$

3. Experimental set-ups

3.1. Acoustic interferometer with amplitude detection of the signal at tuned frequency

Figure 1 shows a block diagram of our two-transducer interferometer with continuous wave at constant measuring distance and variable excitation frequency.

Using this method the amplitude of the standing wave in the specimen of the material immersed in water is observed.

If the frequency of the signal on the emitter transducer (in a given range) is made to vary linearly, successive maxima of signal amplitude on the receiver transducer appear.

At a frequency for which $2l/\lambda$ is an integer, the

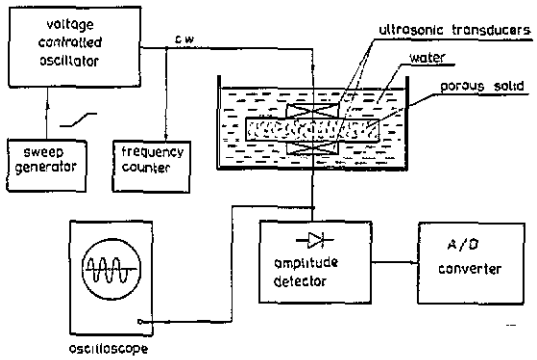

Figure 1. Measuring stand for determining the ultrasonic propagation velocity with amplitude detection in liquid-saturated porous specimens.

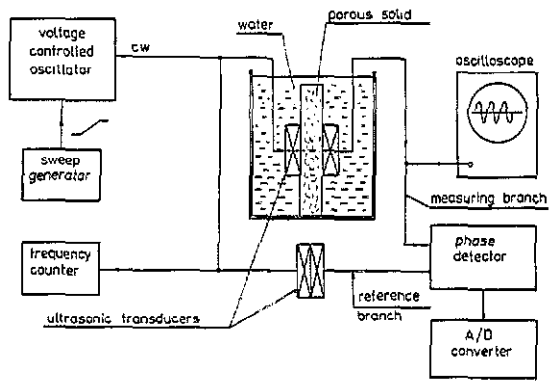

Figure 2. Measuring stand for determining the ultrasonic propagation velocity with phase detection in liquid-saturated porous specimens.

voltage of the signal on the receiver transducer grows steeply due to interference of incident and reflected waves.

On the assumption that the phase velocity of the wave remains constant for change in frequency, measurements of the frequency f_j for successive (labelled by j) maxima of the amplitude enable one to determine the propagation velocity v of the wave in the specimen from the formula [7, 8]

$$v = \frac{2l(f_k - f_n)}{k - n} \tag{5}$$

where k and n label selected peaks of the amplitude.

The 'sweep generator' applied in our measuring circuit in fact produces a voltage growing step-wise with time consisting of 2^N levels (N bits of the digital–analogue converter producing the sweep signal).

Thus, when tuning the generator monitored by the signal (VCO), 2^N discrete frequency values, growing in time, reach the measuring vessel.

On the receiver side, the continuous wave signal is supplied to the amplitude detector. Hence the DC voltage is further transformed (in the analogue–digital converter) to digital form, as required for cooperation with the data-collecting computer.

The absolute error incurred in measuring the propagation velocity of the (dispersionless) wave is dependent on the accuracy with which the length l is measured and on the error in measuring the frequency in the peak (the error in localizing the frequency at resonance), which, in turn, is directly dependent on the number of bits N:

$$\Delta v = \left| \frac{2(f_k - f_n)}{k - n} \Delta l \right| + \left| \frac{l(f_f - f_i)}{2^{N-2}(k - n)} \right| \tag{6}$$

where f_i and f_f are, respectively, the initial and final frequencies within the tuning interval.

3.2. The set-up for determining ultrasonic wave propagation velocity by the phase method with tuned frequency

Figure 2 shows the block diagram of the set-up used for measurements of ultrasonic propagation velocity by the phase method, where the change in phase (in a specimen of constant thickness l) is obtained by varying the frequency of the wave.

Two signals are fed to the phase detector. One is a reference signal for the other signal, coming from the measuring branch.

Along the reference path, containing two mutually contiguous transducers separated by thin copper foil only, serving as earthing for the electric signals, the shift in phase can be evaluated from the formula [10]

$$\varphi_{\text{ref}} = \frac{4\pi f l_T}{v_T} \tag{7}$$

where l_T is the thickness of the transducer, v_T is the propagation velocity of the ultrasonic wave in the transducer, and f is the frequency.

The shift in phase along the measuring branch, φ_s, is augmented by the delay caused by the process of propagation through the medium. Thus

$$\varphi_s = \frac{4\pi f l_T}{v_T} + \frac{2\pi l f}{v} \tag{8}$$

with l the thickness of the specimen, and v the propagation velocity of the ultrasonic wave in the specimen.

Thus, the difference in phase φ between the signals as recorded by the phase detector amounts to

$$\varphi = \varphi_s - \varphi_{\text{ref}} = \frac{2\pi l f}{v}. \tag{9}$$

On the assumption that, in the tuning range Δf, from f_i to f_f the propagation velocity is independent of frequency, the phase detector will show a value $\Delta\varphi$ equal to

$$\Delta\varphi = \frac{2\pi l f_f}{v} - \frac{2\pi l f_i}{v} = \frac{2\pi l(f_f - f_i)}{v} = \frac{2\pi l\,\Delta f}{v}. \tag{10}$$

Hence, the ultrasonic propagation velocity is

$$v = 2\pi l \frac{\Delta f}{\Delta\varphi}. \tag{11}$$

It is worth noting that (besides the absence of velocity dispersion) the precondition for correct measurements

of the velocity is that the relation between increase in phase and increase in frequency shall be linear. This condition is fulfilled only if a running wave propagates in the medium. If there is some slight dispersion, the velocity calculated with equation (9) refers to some intermediate frequency from the interval (f_i, f_f).

The error incurred when measuring the absolute value of the ultrasonic velocity with the formula (11) amounts to

$$\Delta v = \left|\frac{2\pi\,\Delta f}{\Delta\varphi}\Delta l\right| + \left|\frac{2\pi l}{\Delta\varphi}\Delta(\Delta f)\right| + \left|\frac{2\pi l\,\Delta f}{(\Delta\varphi)^2}\Delta\varphi_{\min}\right|. \qquad (12)$$

4. Characteristics of the specimens

In the studies, porous ceramic filters of Pol-Rollit type with porosity of the order of 45% saturated with distilled water were used. Care was taken to ensure their smoothness and parallel configuration of their frontal surfaces. Saturation was achieved by the flow method [5].

The dimensions of the specimens in the direction perpendicular to the front of the wave were 90 mm × 90 mm.

5. Measurements and discussion

When applying the method of amplitude detection of the standing wave arising from interference of transmitted and reflected waves (see figure 1), maxima of the amplitudes on the receiver transducer that appear in the course of frequency tuning are searched for.

Most porous materials exhibit strong attenuation. Thus, strictly, the concept of standing waves in porous specimens is no more than a sort of approximation. Moreover, for some (intermediate) ranges of frequency, phase velocity dispersion is strong, and the attenuation coefficient is by no means constant. The requirements of the method in question are thus, in general, not fulfilled. Nonetheless, experiments show that not too thick liquid saturated porous specimens do in fact exhibit amplitude peaks of standing waves well adapted for determination of wave velocity provided that they involve a region of but slight velocity dispersion.

Figure 3 shows a selected record of the signal amplitude on the receiver head as a function of frequency for specimens 10.9 mm thick, when applying transducers with a resonance frequency of 0.5 MHz. The amplitude peaks appeared, successively, at frequencies $f_1 = 482.6$, $f_2 = 592.8$, $f_3 = 694.5$ and $f_4 = 793.4$ kHz. Almost identical values of the peak frequencies f_2, f_3 and f_4 were obtained using a pair of transducers with a resonance frequency of 1 MHz and measuring the amplitude along the ascending part of the amplitude versus frequency graph (see figure 4).

Good agreement between results obtained along the descending (0.5 MHz head) and ascending part (1 MHz head) of the graph shows that the results are practically

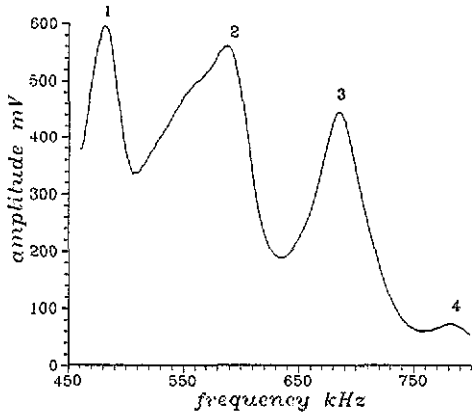

Figure 3. Ultrasonic amplitude graphs on transition of the wave through a specimen 10.9 mm thick, for frequencies ranging from $f_i = 460$ kHz to $f_f = 799$ kHz; (porous material saturated with distilled water; 0.5 MHz heads).

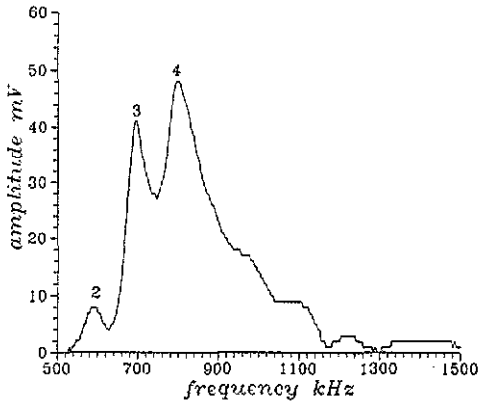

Figure 4. Ultrasonic amplitude graphs on transition of the wave through a specimen 10.9 mm thick, for frequencies ranging from $f_i = 502$ kHz to $f_f = 1502$ kHz; (porous material saturated with distilled water; 1 MHz heads).

unaffected by error in the readings of peak frequencies arising from change in amplitude with varying frequency.

The decreasing differences in frequency between neighbouring peaks $(f_2 - f_1)$, $(f_3 - f_2)$ and $(f_4 - f_3)$ point to a decrease in dispersion of the wave velocity. Equation (5), applied to pairs 2–3 and 3–4 leads to propagation velocities of $v_{2-3} = 2217\ \mathrm{m\,s^{-1}}$ and $v_{3-4} = 2156\ \mathrm{m\,s^{-1}}$, respectively. The slight difference between v_{2-3} and v_{3-4} permits the conclusion that these velocities belong to a frequency range where dispersion of the ultrasonic wave velocity is insignificant.

Equation (6), with the data $f_1 = 482.6$ kHz; $f_4 = 793.4$ kHz; $l = 10.9$ mm; $\Delta l = 50\ \mu$m; $f_i = 460$ kHz; $f_f = 799$ kHz; $N = 8$, corresponding to the experiment (neglecting error due to dispersion or attenuation) permits the following evaluation of the absolute error incurred when measuring the ultrasonic wave velocity:

$$\Delta v = 10.36 + 19.24 = 29.6\ \mathrm{m\,s^{-1}}.$$

A lowering of the second component of the sum in equation (6) can be achieved by use of a digital–analogue converter with greater resolution. For example, for a

12-bit converter the total error might undergo a reduction down to 11.56 m s^{-1}.

With regard to the very strong ultrasonic wave damping in porous materials, the phase detection method discussed in section 3 suggests itself as more useful.

Figure 5 shows changes in phase of waves transmitted by a porous specimen versus frequency for a filter 10.9 mm thick, identical to the one used in our amplitude detection experiment. The frequency was made to vary from 461 to 799 kHz, whereas the change in phase measured amounted to 10.8337 rad.

On the assumption of small velocity dispersion, equation (11) gives a phase velocity of 2140 m s^{-1}.

The difference in velocities given by the two methods thus amounts to several per cent.

The phase graph of figure 5 exhibits local deviations from linearity interpretable as due to reflected waves. This suggestion is corroborated by the fact that considerably more smooth changes in phase are observed for thicker specimens, where the role of reflected waves in phase measurement is less significant. For example, figure 6 shows changes in phase for a filter 31 mm thick saturated with distilled water.

The preceding examples of measurements of change in phase concerned a wide range of variations of frequency.

In the case of waves subject to strong dispersion, the range of frequency tuning should be restricted appropriately. Such measurements, performed in neighbouring ranges of frequency, would enable one to find the approximate dispersion curve and would, moreover, permit determination of the group velocity [7].

On substitution of the following experimental values into equation (12): $\Delta(\Delta f) = 1$ Hz; $\Delta\varphi_{\min} = 0.00153$ rad; $\Delta l = 0.05$ mm; $v = 2140$ m s^{-1}; $\Delta f = 69$ kHz; $\Delta\varphi = 2\pi$ rad; $l = 31$ mm one can arrive at

$$\Delta v = 3.45 + 0.031 + 0.52 \cong 4 \text{ m s}^{-1}$$

for the absolute error incurred in measuring propagation velocity.

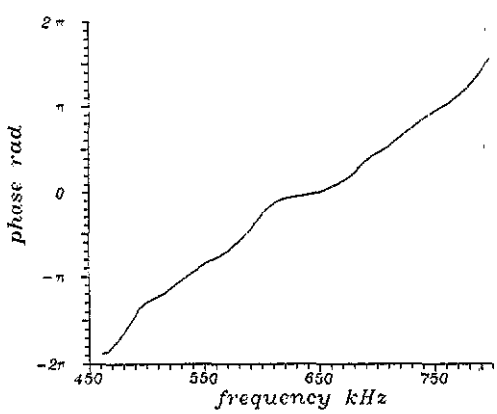

Figure 5. Ultrasonic phase graphs on transition of the wave through a specimen 10.9 mm thick, for frequencies ranging from $f_i = 461$ kHz to $f_f = 799$ kHz; (porous material saturated with distilled water; 0.5 MHz heads).

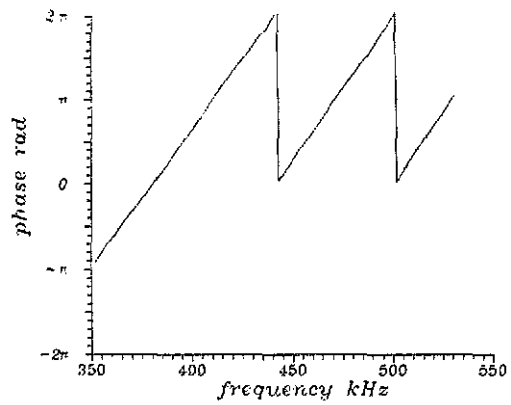

Figure 6. Ultrasonic phase graphs on transition of the wave through a specimen 31 mm thick, for frequencies ranging from $f_i = 351$ kHz to $f_f = 531$ kHz; (porous material saturated with distilled water; 0.5 MHz heads).

The above error is dependent on the thickness of the specimen, the frequency, the properties of the medium, as well as the accuracy in phase measurement. However, it does not comprise inaccuracies due to velocity dispersion and the presence of reflection on the walls of the specimen. In order to eliminate the latter, when studying fluids, one applies special diaphragms [11]. The phase detector used by us in the measuring set-up was one constructed using a digital technique, with ideally linear characteristics.

6. Conclusions

Application of two continuous wave methods involving amplitude detection and, respectively, phase detection, for measurement of the fast wave velocity in saturated porous media was discussed.

The velocity of the fast wave, after Biot (see equation (3)), is a function of frequency, and considerable changes in frequency occur only in a relatively narrow range of intermediate frequencies [2, 3], beyond which the phase velocity of the fast wave varies only slightly with frequency, thus obeying equation (4). This permits the use of continuous wave methods in measurements of the propagation velocity of ultrasonic waves.

In dispersionless regions, these methods ensure a high degree of accuracy. Both the amplitude detection method and the phase detection method enable one to decide whether a given measurement concerns a region with dispersion or a dispersionless region. Narrowing the range of frequency tuning in the phase detection method permits determination (in high dispersion regions) of the phase velocity and group velocity.

Very important are the requirements set by the above techniques when applied to the study of porous materials, regarding the size of specimens and the frequency ranges of the waves.

Experiments applying continuous ultrasonic waves show (compared with the pulse method for example)

that, in saturated porous media, continuous waves with relatively high amplitudes can be excited effectively.

Observations of the continuous wave at the output show no distortion of the sine shape, indicating the absence of a slow wave.

It should be noted that the discussed methods utilize classical laboratory equipment supplemented with essentially improving the technique of the data-collecting computer.

The advantages presented by continuous wave methods in application to study of saturated porous media mean that measurements by these methods are very well adapted to determination of material constants of media.

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