Dynamics of an active magnetic particle in a rotating magnetic field

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Introduction

Active systems interacting with electromagnetic fields [1,2] are interesting from different points of view. The pumping of liquid without force due to negative viscosity effect of dielectric suspensions [3], the flexible magnetic filaments interacting with ac magnetic field [4,5], their selfpropulsion in the liquid [6,7] may also be mentioned here among other examples. Active magnetic systems also exist in nature - the magnetotactic bacteria have a biochemical machinery allowing it to produce ferromagnetic particles inside its body and use them to orientate in the magnetic field of the Earth [8,9].

The study of the behaviour of magnetic particles in a rotating magnetic field has a rather long history (see for example [10] for further references). Among the most recent developments in this field we can mention [11], where the motion of an anisotropic particle in the rotating optical field is investigated.

In spite of this long standing interest in the behaviour of different particles in ac magnetic fields, active particles (self-propelled in a liquid) have never been properly investigated. Here we describe the model of the active magnetic particle proposed recently [12].

Model

The magnetic particle, due to its selfpropulsion, moves with velocity v in the direction of its magnetic moment (this mechanism is used by magnetotactic bacteria for their survival in environment [8,9]). The magnetic moment orientates along the applied field H. The kinetics of orientation is determined by the torque balance acting on the particle

$$-\alpha\frac{d\vartheta}{dt}+MH\sin\beta=0 \tag{1}$$

here M is the dipole moment of the particle, α is the rotational friction coefficient, ϑ is the particle orientation angle with respect to some fixed direction, which we take to be the x axis, and β is the angle between the magnetic field and magnetic moment of the particle.

In the case of a rotating field $\beta=\omega t-\vartheta$ and the equations of the particle motion are as follows

$$\frac{dx}{dt}=v\cos\vartheta \tag{2}$$

$$\frac{dy}{dt}=v\sin\vartheta \tag{3}$$

$$-\alpha\frac{d\vartheta}{dt}+MH\sin\left(\omega t-\vartheta\right)=0 \tag{4}$$

Equation (4) can be put in the form

$$\frac{d\beta}{dt}=\omega-\omega_c\sin\beta \tag{5}$$

here

$$\omega_c=MH/\alpha \tag{6}$$

If $\omega\leq\omega_c$ equation (5) has a stationary solution $\beta=\beta_0$ where $\sin\beta_0=\omega/\omega_c$. It corresponds to motion around a circle. If $\omega>\omega_c$ then the particle can not rotate synchronously with the applied field and angle β is a periodic function of time. A simple integration gives

$$\beta=2\arctan\left(\gamma+\sqrt{1-\gamma^2}\tan\frac{\sqrt{1-\gamma^2}\omega(t-t_0)}{2}\right) \tag{7}$$

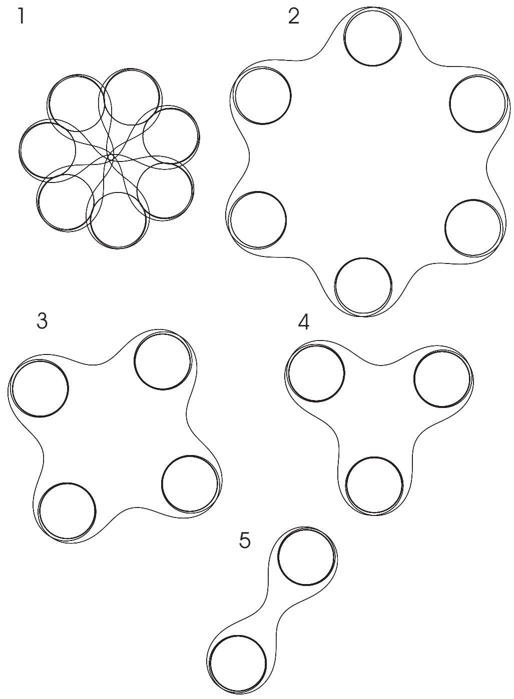

Figure 1: Trajectories of the particle for $\sqrt{1-\gamma^2} = 7/25(1);\ 6/25(2);\ 4/25(3);\ 3/25(4);\ 2/25(5)$.

where $\gamma = \omega_c/\omega < 1$. Since the time moment t_0 can be chosen arbitrarily, we will choose it to be zero. From (7) one can see that the period of the orientational motion is $T_1 = T/\sqrt{1-\gamma^2}$ where $T = 2\pi/\omega$ is the field period. Several trajectories for rational values of ratio $T/T_1 = \sqrt{1-\gamma^2}$ are shown in Fig.1.

Both regimes ($\omega \leq \omega_c$ or $\omega > \omega_c$) are observed experimentally for magnetotactic bacteria [13]. From the figures in [13] one can see that at the frequencies above ω_c the character of bacteria motion changes drastically. Instead of motion along the circles which occurs if $\omega \leq \omega_c$ the bacteria starts to jump between the circles of smaller radius.

Conclusion

It turns out that a simple combination of active properties of a particle with its capability to orientate along the applied field leads to rather rich behaviour which has interesting possibilities for different practical applications among which the determination of the physical properties of the magnetotactic bacteria should be mentioned.

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