Thermomagnetic convection in a central force field

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Introduction

Thermomagnetic convection may be useful to initiate or enhance free convection in Engineering applications[1] but it also offers a way to mimic large-scale convection in planetary or stellar interiors. To simulate this process in the laboratory is problematic as the gravity field in the planetary or stellar interior is radially inwards towards the centre of mass.

Traditional laboratory realisation of a central gravity field is either by a voltage difference across the spherical shell[2] or by placing magnets within the core of the shell[3]. Both result in an effective gravity, $g_e \propto r^{-5}$. However, the gravitational field within a gravitating mass increases with distance from the core. We discuss an arrangement which uses magnets surrounding the sphere[4].

Buoyancy-driven convection is characterised by a Rayleigh number, $Ra = D^3\alpha_0 g\Delta T/(\nu\kappa)$, where D is a length scale, α_0 the volume expansion coefficient, g the gravity, ΔT the imposed temperature contrast, ν the kinematic viscosity, and κ the thermal diffusivity. A similar process is found in a magnetic fluid whose magnetisation, M, decreases with temperature. Thus, a cooler fluid is attracted to the magnet more than a warmer fluid. As such we have a direct equivalent between this force and buoyancy if we replace gravity by the magnetic field gradient and the volume expansion coefficient by the pyromagnetic coefficient. This leads to a corresponding magnetic Rayleigh number,

$$Ra_M = \frac{D^3}{\nu\kappa}\frac{\mu_0}{\rho}\left(\frac{\partial M}{\partial T}\right)_H |\nabla H|\Delta T. \quad (1)$$

The system

The physical system studied here models a small-scale laboratory based experiment of convection in the Earth's core. A sphere with diameter 200mm and radius ratio $R_i/R_o = 0.35$ contains the magnetic fluid, the water-based Ferrotec$^{\text{TM}}$ EMG805 300Gauss magnetic fluid. Using a temperature contrast across the shell of $\Delta T =$ 10K, the system is characterised by a standard Rayleigh number of $Ra = 2.4 \times 10^7$.

The 2D Finite-Element model solved the equations for the magnetic field as well as the heat and momentum equations for the fluid. Unlike all previous studies, we could therefore calculate the actual magnetic field from the permanent magnet, and the resulting magnetisation of the magnetic fluid, rather than imposing an arbitrary or analytical field. The total computation domain was a circle of radius $R_t = 200$. In order to reproduce the convection from the hotter core in the Earth's interior under the linearly increasing but centrally inwards pointing gravity, the inner core was cooled and the outer shell was heated since the magnetic force pointed outwards.

A set of eight magnets, each with a surface magnetisation of 480kAm^{-1} were placed in the outermost shell of the domain as shown in Figure 1, which also shows the resulting magnetic field, $\mathbf{H}$. Solving the field and calculating the corresponding magnetisation and pyromagnetic coefficient, resulted in a radially increasing force field, whose strength is shown by the contour lines in Figure 2. The radial variation of this force field is consistent with a scaling $\propto r^{4.3}$. With a temperature contrast of

$\Delta T = 10K$, the magnetic Rayleigh number increased from $Ra_m = 10^7$ at the inner core to 3×10^8 at the outer boundary. The convection caused by this radially increasing magnetic field is shown in Figure 3. One can see two jets each leaving from the cold core and the hot outer boundary forming fairly regular large-scale convection cells. It was found that this pattern was not very sensitive to the presence or absence of standard vertical gravity in addition to the Kelvin force. Furthermore, this flow was very similar to that obtained by a pure central force field.

Discussion and Conclusions

The results have demonstrated that thermomagnetic convection can be used to simulate a largely radially aligned force field even in the presence of standard gravity. While our system is that of a non-rotating shell, and thus limited in its direct relevance to large-scale geophysical interpretation, it is a first justification for developing this approach further.

References

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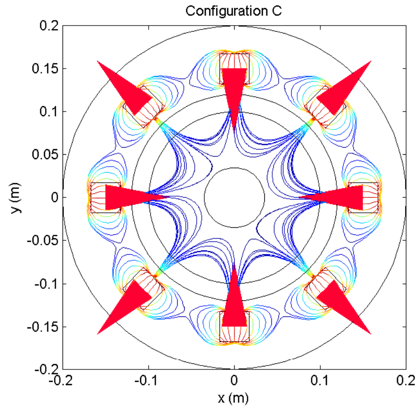

Figure 1. The computational domain showing the orientation of the eight magnets and the magnetic field lines.

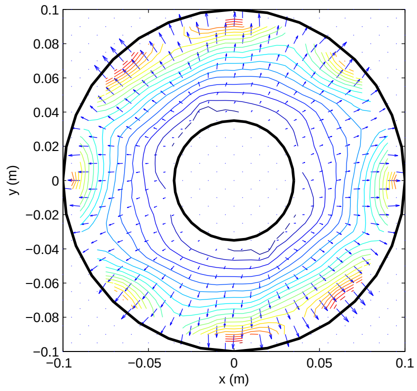

Figure 2. Contour lines of the strength of the Kelvin force.

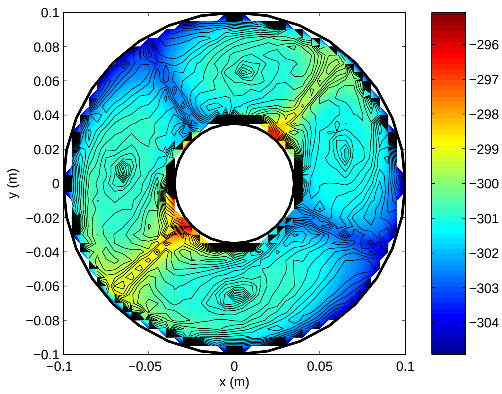

Figure 3. Temperature and vorticity (contour spacing of 0.5s^{-1}).