Slow Magnetosonic Wave Propagating in a Magnetizable Fluids

V. V. Sokolov [1]

[1] *Moscow State Academyof Instrumental Engineering and Computer Science, Department of Mathematics, 20 Stromynka St., Moscow 107846 Russia. vsokolov@mgapi.edu*

The concept of frozen magnetization was used in deducing ferrohydrodynamics equations [1]. It was shown [2] that the fast and slow magnetosonic waves and Alfven-type hydrodynamic wave are the hydrodynamic modes of the ideal non-conducting magnetic fluid with frozen-in magnetization.

The experimental proof of existence of slow magnetosonic wave recently was received in work [3]. The authors of [3] have reported the experimental evidence of fast and slow longitudinal acoustic waves which propagates through a magnetorhelogical slurry composed of hydrogen-reduced spherical iron particles suspended in glycerine. The slow wave has been discussed in the frame of Biot theory which describing the propagation elastic waves in porous fluid-saturated solid [3]. However Brand and Pleiner [4] pointed out on impossibility to describe the slow wave in frame of Biot theory because it propagated in presence of external magnetic field only. Authors of [4] have proposed a different explanation of slow wave based on their theory [5] in which a propagating magnetotlastic sound-like mode due to longitudinal chain vibrations was predicted.

Here we explain these experimental results using the theory of wave propagation in magnetic fluid with frozen-in magnetization.

The complete system of equations describing an ideal non-conducting magnetic fluid of density ρ with the frozen-in magnetization $\vec{M} = \rho\vec{m}$ has the form [1]:

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_k}\left(\rho v_k\right) = 0\,;$$

$$\frac{\partial m_i}{\partial t} + v_k\frac{\partial m_i}{\partial x_k} = m_j\frac{\partial v_i}{\partial x_j}\,;$$

$$\rho\frac{\partial v_i}{\partial t} + \rho v_k\frac{\partial v_i}{\partial x_k} = -\frac{\partial}{\partial x_i}\left(\rho^2\frac{\partial u}{\partial\rho}\right) + \frac{\partial}{\partial x_j}\left(\rho m_j\frac{\partial u}{\partial m_i}\right) - H_i\frac{\partial\left(\rho m_j\right)}{\partial x_j}; \qquad (1)$$

$$H_i = -\frac{\partial\Psi}{\partial x_i}; \qquad \nabla^2\Psi = 4\pi\frac{\partial\left(\rho m_k\right)}{\partial x_k};$$

$$u = u(\rho, s, m_i). \qquad (i, j, k = 1,2,3).$$

The system of equations is closed by setting a specific form of the internal energy per unit mass u, which depends on the fluid density, on the specific entropy s, and on the components of the magnetization per unit mass m_i. The latter two equations of the system (1) are the Maxwell magnetostatic equations, where Ψ is the scalar potential of the magnetic field. Let us investigate the small amplitude waves which propagate through a spatially uni-form magnetic fluid placed in external magnetic field. We can assume, without loss of genera-lity, that the intensity of magnetic field $\vec{H}_0$ is directed along the *z*-axis, and the wave-vector $\vec{k}$ lies in the *y-z* plane and makes an angle ϑ with the *z*-axis. Also we assume that the internal energy density per unit mass has the form

$$u\left(\rho, s, m_i\right) = u_0 + \beta_{\|}m_z^2 + \beta_{\perp}\left(m_x^2 + m_y^2\right),$$

where $\beta_{\|} = \beta_{zz}$, $\beta_{\perp} = \beta_{xx} = \beta_{yy}$ are the components of the tensor $\beta_{ij} = \frac{1}{2}\frac{\partial^2 u}{\partial m_i \partial m_j}$.

We linearize the equations of system (1) with allowance for this explicit form of the functional dependence of the internal energy assuming, for sake of simplicity, that the equilibrium flow velocity is zero.The standard technique is used to solve linearized equations of system (1).

The solutions of the linearized equations determine the velocity of the Alfven-type hydrodynamic wave

$$c_A^2 = m_0^2 \beta_\perp \cos^2 \vartheta$$

the velocity of the fast magnetosonic wave

$$c_s = \frac{c_0}{\sqrt{2}} \sqrt{\left(1 + \frac{c_A^2}{c_0^2}\left(1 + \frac{\beta_\parallel}{\beta_\perp}\right)\right) + \Delta}$$

and the velocity of the slow magnetosonic wave

$$c_s = \frac{c_0}{\sqrt{2}} \sqrt{\left(1 + \frac{c_A^2}{c_0^2}\left(1 + \frac{\beta_\parallel}{\beta_\perp}\right)\right) - \Delta},$$

where

$$\Delta = \sqrt{\left(1 + \frac{c_A^2}{c_0^2}\left(\frac{\beta_\parallel}{\beta_\perp} - 1\right)\right)^2 + 4\frac{c_A^2}{c_0^2}\left(1 - \frac{\beta_\parallel}{\beta_\perp}\right)\sin^2 \vartheta}.$$

Here c_0 denotes the sound propagation velocity in unmagnetized magnetic fluid.

Figure 1 shows the theoretical dependence of the slow wave velocity on the angle values ϑ in magnetorheological suspension investigated in [3].

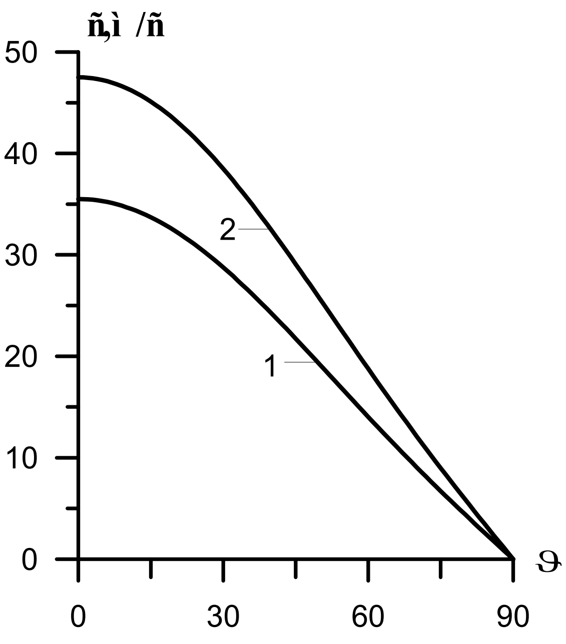

Figure 1.

Curves 1 and 2 describe the anisotropy of the slow wave velocity at $H = 60$ G and $H = 490$ G, respectively. These results can be used in experimental check of the proposed theory. We believe, that slow magnetosonic waves should form interesting area of an experimental research.

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