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Movement of a magnet and a paramagnetic body inside a vessel with a magnetic fluid☆

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Abstract

The trajectories of a spherical magnet and a spherical paramagnetic body (in an applied uniform magnetic field) in a magnetic fluid inside a spherical vessel are calculated numerically. It is shown that the trajectory of the spherical magnet differs essentially from the trajectory of the spherical paramagnetic body.

Keywords: Magnetic force; Moment of the force; Body (magnet) trajectory; A finite magnetic fluid volume

1. Introduction

A phenomenon of the levitation of a permanent magnet which is immersed in a finite volume of a magnetic fluid (MF) has been first discovered by Rosensweig [1]. A calculation of a magnetic force acting on a magnet is a complex problem. The analytic solutions have been obtained for a cylindrical magnet inside a cylindrical vessel [2] and for a spherical magnet inside a spherical vessel (for small magnet displacement from the centre of the vessel) [3]. A magnetic force acts also on a paramagnetic body immersed in MF in an applied uniform magnetic field if MF has the boundary. However, there are no justified analytical results about a force (moment of a force) acting on a body (not a magnet) in a finite volume of MF in an applied uniform magnetic field. Therefore, in this work an analogy between the forces and the absence of an analogy between the moments of the forces that act on a magnet and on a paramagnetic body are discussed. The forces and the moments of the forces that act on a spherical paramagnetic body and a spherical magnet immersed in an MF inside a spherical vessel are calculated. The trajectories of a spherical magnet and a spherical paramagnetic body (in an applied uniform magnetic field) in MF inside a spherical vessel are calculated numerically.

2. Problem of a magnetic force calculation

Suppose a spherical body ($r_{\rm b}$ is a body radius) is at rest inside a homogeneous incompressible MF that fills a spherical vessel ($R_{\rm V}$ is a vessel radius) and let $\boldsymbol{r}_0$ be the body displacement vector. The magnetic permeability of the MF $\mu_{\rm f}$, the body material $\mu_{\rm b}$, and the material surrounding the vessel $\mu_{\rm s}$ are assumed to be homogeneous, where $\mu_{\rm f} = {\rm const} > 1$, $\mu_{\rm b} = {\rm const} \geq 1$, and $\mu_{\rm s} = {\rm const} \geq 1$. The magnetic field is uniform at infinity $\boldsymbol{H} \to \boldsymbol{H}_\infty = {\rm const}$. Gravitation is not taken into account. The force $\boldsymbol{F}_{\rm b}$ acting on the body immersed in MF is $F_{{\rm b}i} = \mu_{\rm f}/4\pi \int_{S_{\rm b}} (H_i H_k - 0.5 H^2 \delta_{ik}) n_k \,{\rm d}S$, where $\boldsymbol{n}$ is an outer normal to the body surface $S_{\rm b}$, $\boldsymbol{H}$ is the magnetic field in MF. Maxwell's equations can be written in the form $\Delta\phi = 0$, $\boldsymbol{H} = \nabla\phi$, $\boldsymbol{B} = \mu\boldsymbol{H}$ in all domains. The potential ϕ and the normal component of the vector $\boldsymbol{B}$ must be continuous on the boundaries and $\nabla\phi \to \boldsymbol{H}_\infty$ at infinity.

In Ref. [4], the formula for the magnetic force acting on the spherical body for small displacement of the body $\varepsilon = r_0/R_{\rm V} \ll 1$ and arbitrary magnetic permeabilities has

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been obtained: $\boldsymbol{F}_b = -C\boldsymbol{r}_0 - C(\boldsymbol{r}_0 \cdot \boldsymbol{H}_\infty)\boldsymbol{H}_\infty/3H_\infty^2$, $C = C(\mu_f, \mu_b, \mu_s, H_\infty, r_b, R_V)$.

3. Analytical calculation of the magnetic force in noninductive approximation

Let us suggest that $(\mu_f - \mu_s)/\mu_s \ll 1$ (noninductive approximation). Consider the vessels of spherical, ellipsoidal, cylindrical shapes or the plane layers (the special shape vessels) with MF. An applied uniform magnetic field creates inside such vessels a uniform magnetic field when the body is absent. We can prove that the force acting on the body in such vessels is calculated by the following formula:

$$\boldsymbol{F}_b = -\frac{\mu_f - \mu_s}{8\pi}\int_{S_V} H_b^2 \boldsymbol{n}\, dS \qquad (1)$$

Here $\boldsymbol{H}_b$ is the distortion of an applied uniform magnetic field owing to the presence of body in the unbounded MF. For the spherical body $\boldsymbol{H}_b$ is the field of a magnetic dipole: $\boldsymbol{H}_b = \nabla((\boldsymbol{m}_b\boldsymbol{r})/r^3)$, $\boldsymbol{m}_b = (\mu_f - \mu_b)r_b^3\boldsymbol{H}_\infty/(\mu_b + 2\mu_f)$.

The well-known formulas for the force and the moment of the force acting on a magnet in a vessel with MF are written as $\boldsymbol{F}_m = -(\mu_f - \mu_s)/8\pi\int_{S_V} H_0^2\boldsymbol{n}\, dS$, $\boldsymbol{M}_m = -(\mu_f - \mu_s)/8\pi\int_{S_V} H_0^2[\boldsymbol{r}\times\boldsymbol{n}]\, dS$. Here $\boldsymbol{H}_0$ is the magnetic field of the magnet in the unbounded MF. These formulas are valid for arbitrary vessels. If $\boldsymbol{H}_0$ is the field of magnetic dipole ($\boldsymbol{H}_0 = \nabla((\boldsymbol{mr})/r^3)$) and $\boldsymbol{m} = \boldsymbol{m}_b$, the magnetic force that acts on a spherical body in the special shape vessels equals the magnetic force acting on the magnet in the same vessels ($\boldsymbol{F}_b = \boldsymbol{F}_m$ if $\boldsymbol{m} = \boldsymbol{m}_b$). The moment of the magnetic force that acts on the spherical body is zero, $\boldsymbol{M}_b = 0$; the moment of the magnetic force acting on the magnetic dipole $\boldsymbol{M}_m$ is not zero, for example, in spherical vessels $\boldsymbol{M}_m = -\boldsymbol{r}_0 \times \boldsymbol{F}_m$. So there is no analogy between the moments of the magnetic forces acting on the body and the magnet.

Using formula (1), we calculate the new formula for the force acting on the spherical body in a spherical vessel for finite displacements:

$$\boldsymbol{F}_b(\boldsymbol{r}_0, \boldsymbol{m}_b) = K_b\left[\left(f_1(r_0) + f_2(r_0)\frac{(\boldsymbol{m}_b\boldsymbol{r}_0)^2}{r_0^2 m_b^2}\right)\frac{\boldsymbol{r}_0}{r_0} + f_3(r_0)(\boldsymbol{m}_b\boldsymbol{r}_0)\frac{\boldsymbol{m}_b}{r_0 m_b^2}\right] \qquad (2)$$

Here $a = r_0/R_V$, $f_1 = 2L + 3N$, $f_2 = 3(2L - 5N - 2aI)$, $f_3 = 6N + 6aI$, $L = -4a(1-a^2)^{-4}$, $I = 4(1-a^2)^{-4}/3$, $N = -1/(8a^4)\ln((1+a)/(1-a)) + ((1+a^2)(3 - 14a^2 + 3a^4))/(12a^3(1-a^2)^4)$, $K_b(m_b) = (\mu_f - \mu_s)m_b^2/8R_V^4$. Using analogy between the magnetic forces acting on the body and the magnet, the new formula for the force acting on a magnetic dipole inside a spherical vessel for arbitrary displacements may be written as $\boldsymbol{F}_m = \boldsymbol{F}_b(\boldsymbol{r}_0, \boldsymbol{m})$, $K_m = K_b(m)$.

Near MF plane boundary (for $\mu_s = 1$) the forces acting on the spherical body and the spherical magnet are $\boldsymbol{F}_{b,m} = -|F_{b,m}|\boldsymbol{n}$ $F_{b,m} = 3(\mu_f - 1)(m_{b,m\,n}^2 + 0.5m_{b,m\,\tau}^2)/16h^4$ where $\boldsymbol{n}$ is external normal to the MF boundary, h is the distance from the body or the magnet centre to the boundary. The moment of the force acting on the spherical magnet is $\boldsymbol{M}_m = -(\mu_f - 1)(\boldsymbol{mn})(\boldsymbol{m}\times\boldsymbol{n})/16h^3$.

4. Trajectories of a magnet and a paramagnetic body in the MF inside a spherical vessel

Let $\boldsymbol{M}$ be the magnet magnetization and r_m be the radius of the magnet. The magnetic moment of the magnet is defined as $\boldsymbol{m} = V_m\boldsymbol{M}$, $V_m = 4\pi r_m^3/3$. If the magnet or the body is not in equilibrium ($\boldsymbol{r}(t=0)\neq 0$) they begin to move; $\boldsymbol{r}$ is the displacement vector of the magnet or the body. Let the initial velocity of the body and the magnet $\boldsymbol{v}(0)$ be zero, initial velocity of rotation of the magnet $\boldsymbol{\omega}(0)$ be zero, initial magnetic moment of the magnet be equal to the magnetic moment of the body ($\boldsymbol{m}(t=0) = \boldsymbol{m}_b = \text{const}$). Let r_b equal r_m, ρ_b equal ρ_m; $r_b = r_m = r$, $V_b = V_m = V$, $\rho_m = \rho_b = \rho$, ρ is a density.

We can prove that the magnet or the body will move on a plane in which vectors $\boldsymbol{m}(t=0)$ and $\boldsymbol{r}(t=0)$ lie. Let the z-axis be perpendicular to this plane. Then following equalities are true $\boldsymbol{m} = (m_x, m_y, 0)$, $\boldsymbol{r} = (x, y, 0)$, $\boldsymbol{\omega} = (0, 0, \omega_z)$, $\boldsymbol{v} = (v_x, v_y, 0)$. Dimensionless parameters are (asterisks denote dimensional parameters, η is the MF viscosity) $\boldsymbol{r} = \boldsymbol{r}^*/R_V$, $\boldsymbol{m} = \boldsymbol{m}^*/m^*$, $t = t^*/t_c$, $t_c = (9\eta/2\rho r^2)^{-1}$, $\boldsymbol{v} = \boldsymbol{v}^* t_c/R_V$, $\boldsymbol{\omega} = \boldsymbol{\omega}^* t_c$, $\boldsymbol{F}_m = \boldsymbol{F}_m^*/K_m$, $\boldsymbol{F}_b = \boldsymbol{F}_b^*/K_b$. The dimensionless equations describing the magnet's motion are

$$\dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{v}_x = -v_x + aF_{mx},$$

$$\dot{v}_y = -v_y + aF_{my}, \quad a = \rho^{-1}V^{-1}t_c^2R_V^{-1}K_m,$$

$$\dot{\omega}_z = -(10/3)\omega_z - b(\boldsymbol{r}^*\times\boldsymbol{F}_m^*)_z, \quad b = 2,5a(R_V/r_m)^2,$$

$$\dot{m}_x = -\omega_z m_y, \quad \dot{m}_y = \omega_z m_x \qquad (3)$$

The dimensionless initial conditions are $\boldsymbol{r}(t=0) = \boldsymbol{r}^0$, $\boldsymbol{v}(t=0) = 0$, $\boldsymbol{\omega}(t=0) = 0$, $\boldsymbol{m}(t=0) = \boldsymbol{m}^0$. The motion of the body is described by the dimensionless equations

$$\dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{v}_x = -v_x + a_bF_{bx},$$

$$\dot{v}_y = -v_y + a_bF_{by}, \quad a_b = \rho^{-1}V^{-1}t_c^2R_V^{-1}K_b. \qquad (4)$$

The initial dimensionless conditions are $\boldsymbol{r}(t=0) = \boldsymbol{r}^0$, $\boldsymbol{v}(t=0) = 0$.

The motion of the magnet is determined by two dimensionless parameters a and b. The motion of the paramagnetic body is determined by a dimensionless parameter a_b. The vector $\boldsymbol{H}_\infty = \text{const}$ is parallel to vector $\boldsymbol{m}^0$. The parameters for which the problem is

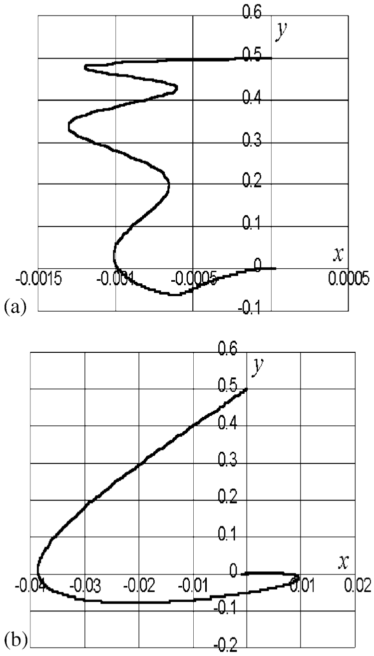

Fig. 1. Trajectories of the spherical magnet (a) and the spherical paramagnetic body (b) for $y^0 = 0.5$.

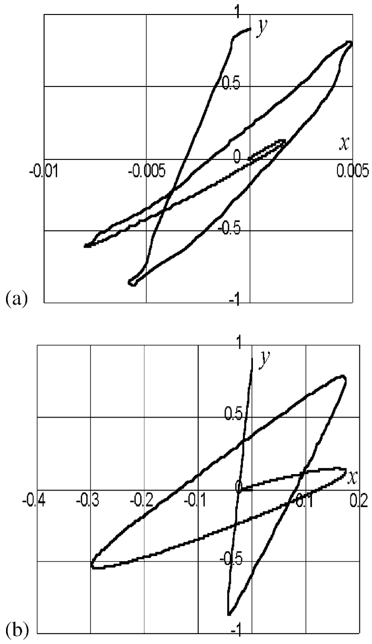

Fig. 2. Trajectories of the spherical magnet (a) and the spherical paramagnetic body (b) for $y^0 = 0.9$.

solved are $R_V = 5\,\mathrm{cm}$, $M = 700\,\mathrm{G}$, $\rho = \rho_b = 5\,\mathrm{g/cm^3}$, $\eta = 0.01\,\mathrm{P}$, $\mu_f = 1.1$, $r_m = r_b = 0.15\,\mathrm{cm}$. All calculations have been made for $a = a_b$. The initial point $\boldsymbol{r}^0 = (x^0, y^0)$, and direction of vector $\boldsymbol{m}^0$ are changed in wide intervals.

For $y^0 = 0.5$, $x^0 = 0$, $m_x^0 = \sqrt{2}/2$, $m_y^0 = \sqrt{2}/2$ the trajectories of the magnet and the body are shown in Fig. 1(a) and (b). In Fig. 2 (a) and (b), the trajectories of the magnet and the body are shown for $y(0) = 0.9$, $x(0) = 0$, $m_x^0 = \sqrt{2}/2$, $m_y^0 = \sqrt{2}/2$. A trajectory of the magnet differs essentially from a trajectory of the paramagnetic body. This fact is related to the rotation of a magnet. The motion of the magnetic moment of the magnet for these two cases are shown in Fig. 3 (a) for $y(0) = 0.5$ and (b) for $y(0) = 0.9$. In Fig. 3, we draw the line $x = m_x(t)/t$, $y = m_y(t)/t$, because $|\boldsymbol{m}| = 1$.

We can show that body displacement along the x-axis is less than the displacement along the y-axis by a factor 10 for a small enough initial displacement. So in real scale, we do not see the move along the x-axis. The body and the magnet move along the initial displacement vector $\boldsymbol{r}^0$. The effective magnetic moment of the body is fixed and the magnetic moment of the magnet changes. It is clear that we can measure the position of the body by any magnetic method because the effective magnetic moment of the body is constant. So a paramagnetic body may be used in the devices that measure motion parameters.

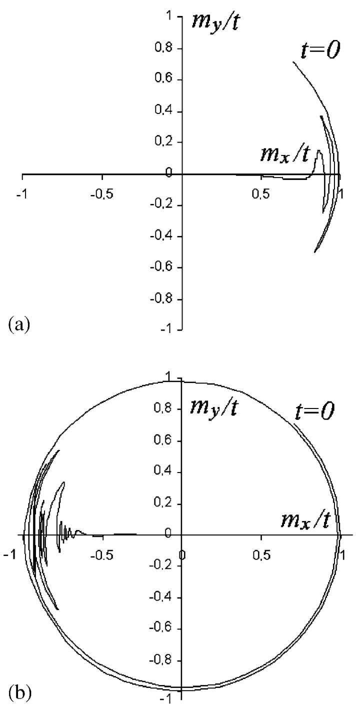

Fig. 3. Direction of vector $\boldsymbol{m}/t$ against t, $t = 0$–$50\,\mathrm{s}$: (a) $y^0 = 0.5$; (b) $y^0 = 0.9$.

5. Conclusions

1. The magnetic force that acts on the body and the magnet are not parallel to displacement vector of the body and the magnet.

2. An analogy between the forces that act on the paramagnetic body and on the magnet exists: in noninductive approximation for spherical body and magnetic dipole in the special form vessels $\boldsymbol{F}_b = \boldsymbol{F}_m$ if $\boldsymbol{m}_b = \boldsymbol{m}$. There is no analogy between the moments of the magnetic forces acting on a body and on a magnet.

3. The trajectory of a magnet differs from the trajectory of a paramagnetic body essentially. This fact is related to a rotation of the magnet and the moment of the magnet.

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