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Thermomagnetic force acting on an ellipsoidal body immersed into a nonuniformly heated magnetic liquid

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Abstract

A prolate spheroidal body immersed into a nonuniformly heated magnetic liquid in an applied magnetic field has been considered. The expressions for the pressure and velocity of the liquid, temperature and magnetic field have been obtained. The formula for a thermomagnetic force acting on the body has been calculated. It has been shown that the body shape needs to be taken into account when we study the thermomagnetic diffusion of the prolate bodies.

Keywords: Magnetic liquid; Magnetic field; Thermomagnetic force

1. Introduction

Many works are devoted to the calculations of the forces acting on a body inside a magnetic fluid in isothermal conditions (see, for example, Refs. [1,2]). In Ref. [3] a thermomagnetic force acting on a spherical body in a nonuniformly heated magnetic fluid has been calculated. Experiments [4] have shown that a thermomagnetic force acts on the large aggregates of ferromagnetic particles in a nonisothermal magnetic liquid. In a magnetic field, the large aggregates have a shape extended along the field. For investigation of the thermomagnetic diffusion of such aggregates it is necessary to know a formula for a thermomagnetic force acting on a prolate body.

Here a prolate spheroidal body immersed into a nonuniformly heated magnetic liquid in an applied magnetic field is considered. The expressions for the pressure and velocity of the liquid, temperature and magnetic field are obtained. These expressions are used for an analytical calculation of the thermomagnetic force acting on the body.

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2. Problem statement

A prolate spheroidal body with major a and minor b axes is considered. The body is immersed into an incompressible viscous nonuniformly heated magnetic liquid in an applied magnetic field. The parameters of the body and the liquid are denoted by the superscripts (i) and (e), respectively. The magnetic permeability of the liquid $\mu^{(e)}$ depends on the temperature T; the magnetic permeability of the body substance $\mu^{(i)}$ depends on the temperature and the magnetic field strength $\boldsymbol{H}$. The thermal conductivity of the liquid and the body substance ($\kappa^{(e)}$ and $\kappa^{(i)}$) are constant. Our aim is to find the thermomagnetic force and this is calculated by the following formula:

$$F^i = \int_S p^{ij} n_j \, \mathrm{d}S, \tag{1}$$

$$p^{ij} = -pg^{ij} + \frac{H^{(e)i}B^{(e)j}}{4\pi} - \frac{\boldsymbol{H}^{(e)}\boldsymbol{B}^{(e)}}{8\pi} g^{ij} + \eta(\nabla^i v^j + \nabla^j v^i),$$

where $\boldsymbol{v}$, p and η are velocity, pressure and viscosity of the fluid, respectively, and $\boldsymbol{n}$ is a normal vector to the body surface S. In Stokes's approximation and for small

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Peclet's number the equations take the forms

$$\operatorname{div} \boldsymbol{B}^{(e,i)} = 0, \quad \operatorname{rot} \boldsymbol{H}^{(e,i)} = 0,$$

$$\boldsymbol{B}^{(e,i)} = \mu^{(e,i)} \boldsymbol{H}^{(e,i)}, \quad \boldsymbol{H} = \nabla \phi,$$

$$\Delta T^{(e,i)} = 0, \quad \operatorname{div} \boldsymbol{v} = 0,$$

$$0 = -\nabla p - \frac{H^2}{8\pi} \nabla \mu^{(e)} + \eta \Delta \boldsymbol{v}.$$

The boundary conditions on the body surface are as follows: $T^{(i)} = T^{(e)}$, $\kappa^{(e)}(\partial/\partial n) T^{(e)} = \kappa^{(i)}(\partial/\partial n) T^{(i)}$, $\boldsymbol{H}_\tau^{(e)} = \boldsymbol{H}_\tau^{(i)}$, $\boldsymbol{B}_n^{(e)} = \boldsymbol{B}_n^{(i)}$, $\boldsymbol{v} = 0$. At infinity, the values perturbed by the body tend to the unperturbed ones noted by the superscripts 0: $T^{(e)} \to T^0$, $\boldsymbol{H}^{(e)} \to \boldsymbol{H}^0$, $\boldsymbol{v} \to 0$.

3. Analytical solution

Let us consider the case when $\boldsymbol{H}_0 \parallel (\nabla T)_0$, $\kappa^{(e)} = \kappa^{(i)}$, $(\nabla_i H_j)_0 = 0$, $i \neq z$, $j \neq z$, $(\nabla_z H_z)_0 \neq 0$ (next formula is valid near the center of the body: $H_j^0 = H_{0j} + (\nabla_i H_j)_0 x^i$). The long axis of the ellipsoid, the axis z and the magnetic field $\boldsymbol{H}_0$ have the same direction.

We introduce the dimensionless values $A^* = A/A_0$, $x_i^* = x_i/a$ ($A \equiv T, \boldsymbol{H}, \boldsymbol{v}, p$, the superscript 0 is related to the unperturbed values at the point coinciding with the body center in the absence of the body) and the small parameter $\varepsilon = a(\nabla T)_0/T_0 \ll 1$.

Under this condition the perturbed values $A'^* = A^* - A^{0*}$ can be found in a power series in ε:

$$A'^* = A_{00} + \varepsilon A_1.$$

The solutions for the dimensionless temperature and the perturbation of the magnetic field potential outside the body are

$$T^{(i),(e)*} = T^{0*},$$

$$\phi^{(e)'*} = \phi_{00}^{(e)} + \varepsilon(\phi_{1\,\mathrm{par}}^{(e)} + A_0 Q_0 + A_2 Q_2 P_2),$$

$$\phi_{1\,\mathrm{par}}^{(e)} = -0.5 \left(\partial \mu^{(e)*} / \partial T^* \right)_0 \phi_{00}^{(e)} z^*,$$

$$\phi_{00}^{(e)} = \frac{c}{a} r_1 M \cos \beta Q_1, \quad r_1 = \left. \frac{\operatorname{ch} \alpha_0}{Q_1} \right|_{\alpha = \alpha_0},$$

$$M = \frac{\mu_0^{(e)} - \mu_0^{(i)}}{\mu_0^{(i)} - \mu_0^{(e)} r_1 \left. \frac{\mathrm{d} Q_1}{\mathrm{d}\,\mathrm{ch}\, \alpha} \right|_{\alpha = \alpha_0}},$$

$$\mu_0^{(i)} = \mu^{(i)}(T_0, (1 + M) \boldsymbol{H}_0),$$

where $c^2 = a^2 - b^2$, $c \operatorname{ch} \alpha_0 = a$; α, β, φ are the ellipsoidal coordinates, $P_i^{(j)}(\cos \beta)$, $Q_i^{(j)}(\operatorname{ch} \alpha)$ are associated Legendre's functions, $Q_i = Q_i^0$, $P_i = P_i^0$. The constants A_0, A_2 are determined from the boundary conditions.

The formulas for the perturbed values of the liquid pressure and velocity have the form (x_i are Cartesian coordinates)

$$\begin{aligned} v_i'^* = {} & \varepsilon Re_m \left(\partial \mu^{(e)*} / \partial T^* \right)_0 \\ & \times (0.5(\phi_{00}^{(e)} + z^*) \phi_{00}^{(e)} \delta_{iz} \\ & + 0.25(x_i^* \phi_{00}^{(e)} - z^{*2} \nabla_i^* \phi_{00}^{(e)}) - u_i) \\ & + 0.5 \varepsilon Re_p x_i^* p^{\mathrm{lap}} + \varepsilon v_i^{\mathrm{lap}}, \end{aligned}$$

$$v_x^{\mathrm{lap}} = c_{21}^x Q_2^{(1)} P_2^{(1)} \cos \varphi + c'^x \frac{c}{a} \frac{\partial Q_1 P_1}{\partial x^*},$$

$$v_y^{\mathrm{lap}} = c_{21}^x Q_2^{(1)} P_2^{(1)} \sin \varphi + c'^x \frac{c}{a} \frac{\partial Q_1 P_1}{\partial y^*},$$

$$v_z^{\mathrm{lap}} = c_0^z Q_0 + c_{20}^z Q_2 P_2 + c'^z \frac{c}{a} \frac{\partial Q_1 P_1}{\partial z^*},$$

$$\begin{aligned} p^* = {} & -\varepsilon P \left(\partial \mu^{(e)*} / \partial T^* \right)_0 \\ & \times (\phi_{00}^{(e)} + z^*) \nabla_z^* \phi_{00}^{(e)} + \varepsilon p^{\mathrm{lap}}, \end{aligned}$$

$$p^{\mathrm{lap}} = a_{01} Q_1 P_1 + a' \frac{\cos \beta}{\operatorname{ch}^2 \alpha - \cos^2 \beta},$$

$$Re_m = \frac{H_0^2 \mu_0 a}{8 \pi \eta v_0}, \quad Re_p = \frac{p_0 a}{\eta v_0}, \ P = \frac{Re_m}{Re_p},$$

where $u_i = \nabla_i^* f$, f is a partial solution of the equation $\Delta^* f = \phi_{00}^{(e)} \nabla_z^* \phi_{00}^{(e)}$. The coefficients a_{01}, a', c_0^z, c_{20}^z, c'^z, c_{21}^x, c'^x are found from the boundary condition for the velocity and from the equation $\operatorname{div} \boldsymbol{v} = 0$.

With the help of these solutions and Eq. (1) the thermomagnetic force is obtained:

$$\begin{aligned} F_z = {} & \frac{V}{4\pi} \left(\frac{\partial \mu^{(e)}}{\partial z} \right)_0 H_0^2 \frac{1}{Q_1 \operatorname{sh}^2 \alpha_0} \\ & \times \left[\left(\frac{3}{4} + \frac{1}{2} \frac{Q_2 \operatorname{ch}^2 \alpha_0}{Q_1 \operatorname{ch} \alpha_0 + Q_0} \right) M \right. \\ & \left. + \frac{1}{2} M^2 \frac{Q_1 \operatorname{ch} \alpha_0}{Q_1 \operatorname{ch} \alpha_0 + Q_0} \right], \end{aligned} \tag{2}$$

where $V = 4\pi a b^2/3$ is the body volume, $Q_i = Q_i(\operatorname{ch} \alpha_0)$. The force has only one component along z-axis. Analysis of formula (2) shows that the force increases with the parameter $s = a/b$ when the body volume is constant. For large enough parameter s ($s \to \infty$) the formula for the force may be written as

$$\begin{aligned} F_z = {} & \frac{V}{4\pi} \left(\frac{\partial \mu^{(e)}}{\partial z} \right)_0 H_0^2 \left(M + \frac{M^2}{4} \right) \\ & \times \frac{s^2}{\ln s} \left[1 + O\left(\frac{1}{\ln s} \right) \right]. \end{aligned} \tag{3}$$

We can establish that for $M = -1$ and for the parameter $s = 10$ the force acting on the extended spheroidal body of volume V is about 10 times greater than the force acting on the spherical body with the same volume.

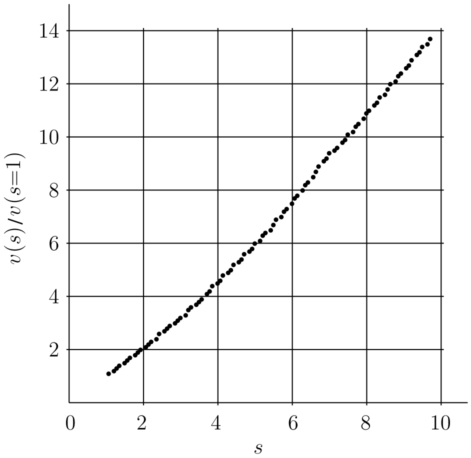

Fig. 1. Value of $v(s)/v(s=1)$ against s (for $M=-1$).

Owing to the presence of the thermomagnetic force a body begins to move in a viscosity liquid. The friction force acting on the moving bodies in a viscosity liquid equals $6\pi\eta R_{\text{eff}}v$, where η is viscosity, $R_{\text{eff}} = R_{\text{eff}}(V, s)$ is the effective radius of the extended spheroidal body, and v is velocity of the body. For small enough body, the body velocity may be found from the equation $6\pi\eta R_{\text{eff}}v = F_z$. Using information about $R_{\text{eff}} = R_{\text{eff}}(V, s)$ from Ref. [5] and formula (2) we can calculate the extended spheroidal body velocity $v(s, V, M)$ and the ratio $v(s, V, M)/v(s=1, V, M) = f(s, M)$. In Fig. 1 the ratio $v(s)/v(s=1)$ against s is shown for $M=-1$. This curve $v(s)/v(s=1) = f(s)$ shows that the velocity of extended spheroidal body may be considerably more than the velocity of the spherical body with the same volume.

4. Conclusions

Using formulas (2) and (3), the following conclusions may be obtained:

(1) Vector of the thermomagnetic force and vector ∇T have the same direction if $M<0$ and opposite direction if $M>0$. It means that the prolate aggregates of the ferromagnetic particles move to more heated boundary of a magnetic liquid.
(2) The velocity of an extended spheroidal body in a nonuniformly heated magnetic liquid depend heavily on the parameter $s = a/b$. So, body shape needs to be taken into account when we estimate the aggregate velocity caused by the thermomagnetic force.

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