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Waves on the surface of a magnetic fluid layer in a traveling magnetic field

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Abstract

The plane flow of a layer of incompressible viscous magnetic fluid with constant magnetic permeability under the action of a traveling magnetic field is analyzed. The strength of the magnetic field producing a sinusoidal traveling small-amplitude wave on the surface of a magnetic fluid is found. This flow can be used in designing mobile robots.

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1. Introduction

The possibility of creating the viscous magnetic fluid flow by means of a magnetic field is investigated. This effect can be used in designing autonomous mobile robots without a hard cover. Such robots can be employed in clinical practice and biological investigations. In Ref. [1], the flow of a viscous fluid layer due to an undulation of bounding impermeable walls was considered. The undulatory perturbations of the wall surfaces and the fluid velocity at the wall were assumed to be given. The average flow rate in the layer was calculated for the boundary moving as a sinusoidal traveling wave and the boundary velocity perpendicular to the unperturbed boundaries. In Ref. [2], an analogous flow was considered taking into account the influence of the adjacent layer of another viscous fluid. The undulatory perturbation of the interface between the two viscous fluid layers was given. For the boundary perturbed in the form of a sinusoidal traveling wave, the average flow rate in the layer was calculated. In Refs. [3,4], the behavior of a magnetic fluid film on a rotating horizontal disk in a non-uniform magnetic field was studied. It was shown that the magnetic field affects the film shape considerably, for example, turning on the field leads to the formation of a moving layer thickness jump. In Ref. [5], the motion of a magnetic fluid layer in a traveling magnetic field was investigated experimentally. A dependence of the surface velocity on the magnetic field amplitude and frequency and

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the layer thickness was revealed. In that paper, the theoretical analysis was performed within the framework of the ideal fluid model. In the present study, the motion of an incompressible magnetic fluid layer on an undeformable substrate, produced by a non-uniform magnetic field, is analyzed within the framework of the viscous fluid model. The strength of the magnetic field producing a prescribed surface shape, in particular, the traveling sinusoidal wave, and the velocities, including those at the surface, are to be found. The dependence of the magnitude of the magnetic field on the wavelength is analyzed and the average flow rate is calculated.

2. Formulation of the problem

We will consider plane flow of an incompressible viscous magnetic fluid layer on a horizontal surface in a non-uniform alternating magnetic field (Fig. 1). The magnetic permeability of the fluid μ is assumed to be constant. Here we do not consider effects of finite time of magnetization relaxation τ because $\tau/t_{\mathrm{H}} \ll 1$ (t_{H} is a characteristic time of a field alteration). The environment is unmagnetizable and the pressure on the free fluid surface is constant. In the case of constant magnetic permeability, the body magnetic force is absent and the magnetic field manifests itself in a surface force acting on the free surface [6]. The gravity is not taken into account.

In this case, the system of equations consists of the continuity and Navier–Stokes equations:

$$\operatorname{div} \boldsymbol{v} = 0,$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\operatorname{grad} p + v\Delta \boldsymbol{v}.$$

Here, $\boldsymbol{v} = (u, w)$ and p are the velocity vector and the fluid pressure, $v = \eta/\rho$ and η are the kinematic and dynamic fluid viscosity coefficients, and ρ is the fluid density. The boundary conditions have the form

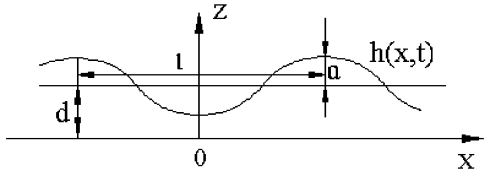

Fig. 1. Magnetic fluid layer.

$$z = 0: \quad \boldsymbol{v} = 0, \tag{1}$$

$$z = h: \quad \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} = w, \tag{2}$$

$$\left[-p + \frac{\gamma}{R} + \frac{B_n^2}{8\pi}\left(\frac{1}{\mu} - 1\right) - \frac{H_\tau^2}{8\pi}(\mu - 1)\right]\boldsymbol{n} + \tau_{ij}n^j e^i = 0. \tag{3}$$

Here, τ_{ij} are the viscous stress tensor components, R is the radius of curvature of the line $z = h(x, t)$, $\boldsymbol{n}$ is the vector of outward normal to the surface, $\boldsymbol{e}^j$ are the basis vectors, γ is the surface tension coefficient, $B_{\mathrm{n}} = \mu H_{\mathrm{n}}$ is the normal component of the magnetic induction vector, and $\boldsymbol{H}_\tau$ is the tangential component of the magnetic field strength vector. The magnetic field $\boldsymbol{H}$ is assumed to be fixed, since the non-inductive approximation $\mu - 1 \ll 1$ is considered. The surface magnetic force is then equal to $(\mu - 1)H^2(x, z = h(x, t), t)\boldsymbol{n}/8\pi$.

We will assume that the magnetic field produces a traveling periodic wave on the surface of a sufficiently thin magnetic fluid layer:

$$z = h(x, t) = d + a\cos(\omega t - kx), \quad \mathrm{d}k = \varepsilon \ll 1.$$

We introduce the following dimensionless variables (the dimensional variables are here denoted with the asterisk):

$$x = x^*k, \quad z = \frac{z^*}{d}, \quad h = \frac{h^*}{d}, \quad u = \frac{u^*}{U_c}, \quad w = \frac{w^*}{\varepsilon U_c},$$
$$U_c = \frac{\omega}{k}, \quad \delta = \frac{a}{d}, \quad t = t^*\omega, \quad p = \frac{p^*}{P}, \quad P = \frac{\eta\omega}{\varepsilon^2},$$
$$H^2 = \frac{H^{*2}}{P}, \quad \mathrm{Re} = \frac{\rho U_c d}{\eta}, \quad W = \frac{\gamma d k^2}{P}.$$

We can simplify the dimensionless equations and boundary conditions (1)–(3) similar to Refs. [3,7] and in the zeroth approximation ($\varepsilon = \mathrm{d}k \to 0$) obtain the following system of equations and boundary conditions for $W = O(1)$ and $\mathrm{Re} \lesssim 1$:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{4}$$

$$-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial z^2} = 0, \quad \frac{\partial p}{\partial z} = 0. \tag{5}$$

$$\text{For } z = h: \quad \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} - w = 0,$$

$$\frac{\partial u}{\partial z} = 0,$$

$$p(x,t) = -W\frac{\partial^2 h}{\partial x^2} - \frac{H^2(x, z=0, t)}{8\pi}(\mu - 1).$$

$$\text{For } z = 0: \quad u = w = 0. \tag{6}$$

Here, for a small layer thickness h, $H^2(x, z = h, t)$ can be assumed to be equal to $H^2(x,t) = H^2(x, z = 0, t)$. Such assumption allow us to simplify the boundary conditions. From Eqs. (4) and (5) and boundary conditions (6) we obtain the relations for the velocity components

$$u(x,z,t) = F(x,t)\left(\frac{z^2}{2} - hz\right), \quad F(x,t) = \frac{\partial p}{\partial x}, \tag{7}$$

$$w(x,z,t) = F(x,t)\frac{\partial h}{\partial x}\frac{z^2}{2} + \frac{\partial F}{\partial x}\left(\frac{h}{2}\frac{z^2}{2} - \frac{z^3}{6}\right). \tag{8}$$

From relations (6)–(8) we derive the equation for h (our approach is similar to shallow water approximation on magnetohydrodynamics [8,9])

$$\frac{\partial h}{\partial t} = \frac{\partial F h^3/3}{\partial x}. \tag{9}$$

From the mass conservation law and Eq. (9) it follows the equation

$$\frac{\partial h}{\partial t} = -\frac{\partial Q}{\partial x}, \quad Q(x,t) = \int_0^h u(x,z,t)\,\mathrm{d}z = -Fh^3/3. \tag{10}$$

For $h = h(\xi)$ and $\xi = t - x$ (in the dimensionless form) it follows from Eqs. (10) that for an arbitrary ε the volume flow rate Q is equal to

$$Q(\xi) = h(\xi) + C, \quad C = \text{const}. \tag{11}$$

In the case of periodic motion, we can introduce the flow rate $\bar{Q}(x)$ average over the period T: $\bar{Q}(x) = \int_0^T Q(x,t)\,\mathrm{d}t/T$. If $h(\xi) = 1 + \delta\cos(\xi)$, the dimensionless average flow rate $\bar{Q}$ is equal to $1 + C$. We note that this flow with non-zero flow rate can be used for designing autonomous movers without hard walls.

If $h = h(\xi)$, with account for Eqs. (11) and (10), the equation for the magnetic field strength takes the form

$$F = W\frac{\partial^3 h}{\partial \xi^3} + \frac{(\mu - 1)\partial H^2}{8\pi \quad \partial \xi} = -\frac{3}{h^2} - \frac{3C}{h^3}. \tag{12}$$

3. Solution of the problem in the case of small amplitude of surface oscillations

Let $h = 1 + \delta\cos\xi$, where $\delta \ll 1$ (the dimensionless amplitude is small). With Eq. (12) taken into account, we represent F as a series in powers of the parameter δ:

$$\begin{aligned} F = &-3\big(1 + C + \tfrac{3}{2}\delta^2 + 3C\delta^2 + \tfrac{15}{8}\delta^4 + \tfrac{45}{8}C\delta^4 \\ &- (2 + 3C)\delta\cos\xi + 3\big(\tfrac{1}{2} + C\big)\delta^2\cos 2\xi \\ &- (4 + 10C)\delta^3\cos^3\xi \\ &+ 5(1 + 3C)\delta^4\big(\tfrac{1}{2}\cos 2\xi + \tfrac{1}{8}\cos 4\xi\big)\big). \end{aligned} \tag{13}$$

Here we assume that $\delta^5 > \varepsilon$. Correct to terms of the order of δ^5, the gradient of the square of the magnetic field strength is equal to

$$\frac{\partial H^2}{\partial \xi} = K(C,\ \delta) + f(\xi),$$

$$K = -\frac{3 \times 8\pi}{\mu - 1}\big(1 + C + \tfrac{3}{2}\delta^2 + 3C\delta^2 + \tfrac{15}{8}\delta^4 + \tfrac{45}{8}C\delta^4\big),$$

$$\begin{aligned} f(\xi) = &-W\frac{8\pi\delta}{\mu - 1}\sin\xi - \frac{3 \times 8\pi}{\mu - 1} \\ &\times \big(-(2 + 3C)\delta\cos\xi + 3\big(\tfrac{1}{2} + C\big)\delta^2\cos 2\xi \\ &- (4 + 10C)\delta^3\cos^3\xi + 5(1 + 3C)\delta^4 \\ &\times \big(\tfrac{1}{2}\cos 2\xi + \tfrac{1}{8}\cos 4\xi\big)\big). \end{aligned} \tag{14}$$

From Eq. (14) we can see that $K = \text{const}$, $\int_0^{2\pi} f\,\mathrm{d}\xi = 0$. Our aim is to find periodical square of the magnetic field H^2 (for $z = 0$). The periodical solution realizes for $K = 0$ only, the constant C can then be found

$$C = -\frac{1 + \frac{3}{2}\delta^2 + \frac{15}{8}\delta^4}{1 + 3\delta^2 + \frac{45}{8}\delta^4}.$$

Expanding the expression for C in a series in powers of δ, we obtain the following expression for the dimensionless average flow rate $\bar{Q}(x)$:

$$\bar{Q}(x) = 1 + C = \tfrac{3}{2}\delta^2 - \tfrac{3}{4}\delta^4 + O(\delta^6).$$

The dimensional flow rate is equal to

$$\bar{Q}^*(x) = d\frac{\omega}{k}\bar{Q}(x) = d\frac{\omega}{k}\left(\tfrac{3}{2}\delta^2 - \tfrac{3}{4}\delta^4 + O(\delta^6)\right).$$

Integrating equality (14) with account for the assumptions made, we obtain the relation

$$\begin{aligned} H^2 &= H_0^2 - D, \\ D &= \frac{8\pi}{\mu - 1}\left[\delta(-W\cos\xi + 3\sin\xi) + \delta^2\left(-\tfrac{9}{4}\sin 2\xi\right) + \delta^3\left(\tfrac{3}{2}\sin 3\xi\right) + \delta^4\left(-\tfrac{3}{4}\sin 2\xi - \tfrac{15}{16}\sin 4\xi\right)\right]. \end{aligned} \quad (15)$$

We choose the constant H_0 arbitrary, greater or equal to $D_{\max}$, since $H^2 \geqslant 0$ and $\int_0^{2\pi} D\,\mathrm{d}\xi = 0$. In what follows, we will assume $H_0 = D_{\max}$. We express the value of magnetic field in terms of the dimensional variables: $H^{*2} = \eta\omega H^2/\varepsilon^2$. For calculating the dimensional maximum magnitude of the magnetic field $H^*_{\max}$, we take the following parameter values: $\eta = 0.1$ P, $d = 0.1$ cm, $\omega = 0.1\ \mathrm{s}^{-1}$, $\delta = 0.1$, $\gamma = 10^2\ \mathrm{D\,cm}^{-1}$, $\rho = 1\ \mathrm{g\,cm}^{-3}$, $\mu = 1.1$. In Fig. 2, the dependence of $H^*_{\max}$ (Oe) on the wave vector is presented. We note that the wave vector is bounded below by the requirement $\mathrm{Re} \leqslant 1$, which is, for the above-specified parameters, reduced to $k \geqslant 0.1$. From Fig. 2, it can be seen that for $k = 0.7$ there exists a minimum of the maximum magnitude of the magnetic field. Fig. 3 shows the spatial distribution (at a fixed time) of the magnitude of the magnetic field strength for different k and the above-specified parameter values.

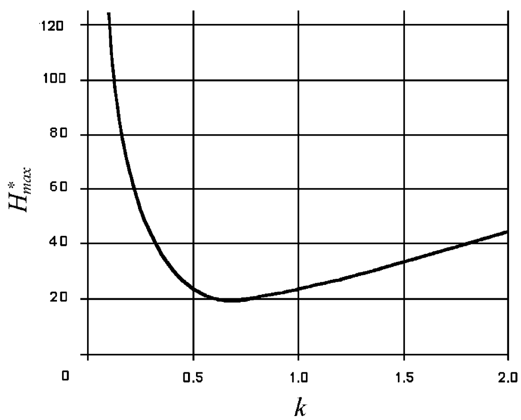

Fig. 2. Dependence of $H^*_{\max}$ on k.

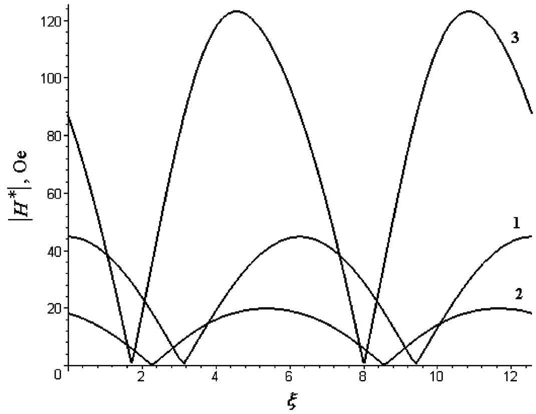

Fig. 3. Magnitude of the magnetic field strength $|H^*|$ as a function of the ξ coordinate. $1 - k = 2$; $2 - k = 0.7$; $3 - k = 0.1$.

The sign of the magnetic field may alternate but so that its absolute value satisfies formula (15). For the parameter values used, the average volume flow-rate per unit channel thickness is equal to $\bar{Q}^* = (0.00015/k)\ \mathrm{cm}^3\,\mathrm{s}^{-1}$. We can anticipate that in the magnetic field whose gradient changes according to the found law a mobile robot with free magnetic fluid surface bounded by an elastic film will move opposite to the traveling wave.

If δ is sufficiently small, the magnitude of the magnetic field can be represented in the form ($\tan\phi = 3/W$):

$$H^2 = H_0^2 - \frac{8\pi}{\mu - 1}\delta\sqrt{W^2 + 9}\cos(\xi + \phi) \quad (16)$$

Let $H_0^2 = 8\pi\delta\sqrt{W^2 + 9}/(\mu - 1)$. Then the absolute value of the magnetic field is equal to

$$H = H_a^*\left|\sin\frac{\xi + \phi}{2}\right|, \quad (17)$$

$$H_a^* = 4\sqrt{\frac{\pi\delta}{\mu - 1}\sqrt{\gamma^2 d^2 k^4 + 9\frac{(\omega\eta)^2}{d^4 k^4}}} \quad (18)$$

From Eq. (18) it is obvious that at a certain value $k = k_{\min}$ the amplitude of the magnetic field reaches its minimum $k^8_{\min} = 9(\eta\omega)^2/\gamma^2 d^6$. The restrictions on the parameters at which this minimum can be reached in the approximation considered ($\varepsilon \ll 1$) and on the frequency in this approximation ($\varepsilon \mathrm{Re} \ll 1$) have the form: $\omega \ll \gamma/3\eta d$, $\omega \ll \eta/d^2\rho$. We note that the inequality $\varepsilon \mathrm{Re} \ll 1$ does not restrict the wavelength but essentially bounds the frequency above: $\omega \ll \eta/(d^2\rho)$. If we like to obtain a flow with non-zero flow rate at low magnetic fields, we must choose the wave vector equal to $k_{\min}$ for which the necessary amplitude is minimal. For example, if $\eta = 1\,\mathrm{P}$, $d = 0.1\,\mathrm{cm}$, $k = 1\,\mathrm{cm}^{-1}$, $\omega = 10\,\mathrm{s}^{-1}$, $\gamma = 3 \times 10^4\,\mathrm{D\,cm}^{-1}$, $\rho = 1\,\mathrm{g/cm}^{-3}$, $\mu = 1.1$, and $\delta = 0.1$, the average flow rate equals $\bar{Q}^* = (1.5 \times 10^{-2}/k)\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ and $k = k_{\min} = 1$. In this case, the amplitude of the magnetic field is minimal and in the linear approximation equal to 461.8 Oe. The condition $\varepsilon \ll 1$ restricts k ($k \ll 10$) and the wave vector value $k = k_{\min} = 1$ is allowable (the condition $\varepsilon \ll 1$ is satisfied). With decrease in the magnitude of the wave vector ($k < 1$) for all the other parameters fixed, the flow rate increases but the amplitude of the magnetic field producing this flow rate also increases significantly.

4. Conclusion

1. An expression is obtained for the strength of the magnetic field producing a sinusoidal wave on the surface of a viscous magnetic fluid as a function of the characteristics of the fluid (viscosity, surface tension, and magnetic permeability) and the parameters of the wave (amplitude, frequency, and wave number). It is shown that at a certain wavelength the maximum magnitude of the magnetic field has a minimum.
2. The average flow rate produced by this magnetic field is calculated. The dependence of the average flow rate on the layer thickness and the amplitude, frequency and length of the surface wave is found.
3. The dependencies obtained can be used as a basis for designing mobile robots in the form of envelopes filled with magnetic fluid.

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