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LETTER TO THE EDITOR

The magnetic susceptibility of chain structures of high-T_c superconductive particles dispersed in magnetic fluids

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Abstract. The magnetic susceptibility of chain and disordered structures of high-T_c superconductive particles dispersed in textured polymerised magnetic fluids has been calculated by means of the Maxwell–Garnet theory using an iterative procedure. The critical magnetic fields, H_{c1}, for various structures are compared with the critical magnetic field H_{c1} of the bulk.

A recent development in the use of magnetic fluids has been in the preparation of magnetic fluid composites [1]. A magnetic fluid composite is a dispersion of non-magnetic particles such as polystyrene, copper, tin and so on of approximately 1–10 μm in diameter, in the magnetic fluid itself. Skjeltorp [2] has shown that in a magnetic fluid, non-magnetic particles appear to acquire an induced magnetic moment which depends on the volume of particles, the properties of the magnetic fluid and on the applied external magnetic field. The non-magnetic particles will thus behave as a many body system with dipolar interactions which may be influenced by an external field. Hence, various many body effects, order–disorder phenomena such as melting, crystallisation, aggregation and so on have been investigated from microscopic observations in magnetic fluids [2].

The discovery of high-T_c superconductive materials with a critical temperature, T_c, above nitrogen temperature has made it possible to study superdiamagnetic fluids [3] which have interesting physical properties.

These fluids are analogous to the traditional magnetic fluids because high-T_c superconductive particles dispersed in liquid nitrogen for example have a magnetic moment due to the Meissner effect.

Another possibility is to study the magnetic fluid composites with dispersed high-T_c superconductive particles. By means of the concept of magnetic holes in the Skjeltorp sense some chain and cluster structures of high-T_c superconductive particles can be established. Using the polymerisation technique in magnetic fluids, solid chain–cluster structures can in principle be established as is shown in paper [4]. Recently Choy and Stoneham [5] have used the effective medium theory for the calculation of some transport, electromagnetic and thermodynamic properties of granular high-T_c superconductors. For the present work we have used the Maxwell–Garnet theory [6] for the calculation of effective susceptibility of chain and disordered structures of high-T_c particles dispersed in polymerised magnetic fluid.

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We shall assume that the chain structures are equidistantly ordered (see figure 1) and we shall neglect the interaction between them.

The magnetic field will be presented below in the form of a dimensionless parameter $\xi = \mu_0 mH/kT$ where m is the magnetic moment of the single magnetic particle. In the calculations we must initially assume a susceptibility of the mixture components, a susceptibility of the magnetic fluid and a susceptibility of the superconductive spherical particles respectively.

Briefly we can say that a textured ferrofluid arises when a magnetic field is applied on a magnetic fluid with a polymerised basic fluid medium.

According to the principle of minimum free energy the axis of easy magnetisation of each magnetic particle has been distributed around the direction of the polymerisation field. The solid polymer prohibits particles and then the magnetic fluid keeps its anisotropic properties. We then define, according to the work in [7], the phenomenological parameter S, which is a number from 0 to 1 ($S = 0$ for untextured and $S = 1$ for an ideal textured magnetic fluid). This parameter characterises the anisotropy of a magnetic fluid.

Our assumption for matrix magnetic fluid susceptibility will then be given by the following equations:

$$\chi_{Fk} = \chi_{0k}[1 + S(3\delta_{1k} - 1)] \tag{1}$$

and

$$\chi_{0k} = 3\chi_0 L(\xi_{Fk})/\xi_{Fk} \tag{2}$$

where k is an index which denotes the direction of the main axes (see figure 1), $k = 1$ denotes the direction of the polymerisation field, δ_{1k} is the Kronecker symbol, χ_0 is the initial susceptibility of the magnetic fluid, ξ_{Fk} is the field in the magnetic fluid component of the mixture and $L(x)$ is the Langevin function. We assume that the direction of the texture of magnetic fluid particles is the same as the direction of the chains. The equations (1) and (2) correspond with [7] for $\xi_{Fk} \rightarrow 0$.

For superconductive medium of the second kind we shall assume the following phenomenological equation [8]

$$\chi_{Sk} = -\,\theta(\xi_{c1} - \xi_{Sk}) - g_m\theta(\xi_{Sk} - \xi_{c1})\theta(\xi_{c2} - \xi_{Sk}) \tag{3}$$

where g_m is the mixed state curve which shall be modelled by the function

$$g_m = [\xi_{c1}/(\xi_{c2} - \xi_{c1})]\xi_{c1}\xi_{c2}/\xi_{Sk}^2 - \xi_{c1}/\xi_{Sk}) \tag{4}$$

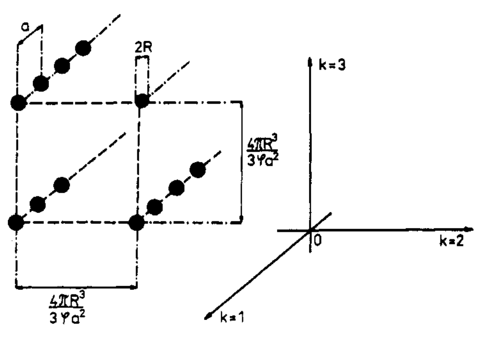

Figure 1. The structure of high-T_c superconductive particles used in the calculations. k denotes the direction of the axes.

where

$$\theta(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ 0 & \text{for } x < 0. \end{cases} \tag{5}$$

In equation (4) ξ_{c1} and ξ_{c2} correspond to the critical magnetic fields H_{c1} and H_{c2} and ξ_{Sk} is the mean field parameter in the superconductivity particle. The case of equation (4) corresponds with the results obtained for perovskite [9] if we neglect the inverse quadratic member, which is added to fulfil the conditions: $\chi_{Sk}(\xi_{c1}) = -1$ and $\chi_{Sk}(\xi_{c2}) = 0$.

Symmetry between different quantities in electromagnetic theory enable us to use the results for electric permeability in the case of magnetic susceptibility.

For mean susceptibility χ_{Mk} in the sample we can write the equation [10]

$$\chi_{Mk} = \{3\varphi f_k + \chi_{Fk}[1 + f_k(3\varphi - X_k)]\}/(1 - X_k f_k) \tag{6}$$

where

$$f_k = (\chi_{Sk} - \chi_{Fk})/(2\chi_{Fk} + \chi_{Sk} + 3) \tag{7}$$

φ is the volume fraction of superconductive particles and the meaning of symbols X_k for the main directions and distinct structures is given in table 1.

If we include in the results demagnetising factors D_k ($D_1 \simeq D_2 < D_3 \, \Sigma_k \, D_k = 1$) we can write the following relation for the effective susceptibility χ_{Ek}

$$\chi_{Ek} = \frac{\chi_{Mk}}{1 + D_k \chi_{Mk}}. \tag{8}$$

Table 1. The meaning of symbols X_k in equation (6).

$\sum_1 (3) = \sum_{i=1}^{\infty} 1/i^3 = 1.202$

$\sum_{2,3} (3) = \sum_{h=0}^{\infty} \sum_{i=1}^{\infty} \frac{1}{(h^2 + i^2)^{3/2}} = 2.212$

for infinite chains; k denotes the direction of the axes.

Structure of superconductive particles	Structure of magnetic fluid	Direction	Meaning of X_k
Disorder	Without texture $S = 0$	$k = 1, 2, 3$	$X_k = \varphi$
Chains	$0 < S \leq 1$	Parallel to polymerisation field, $k = 1$.	$X_1 = 4\left(\frac{R}{a}\right)^3 \sum_1 (3)$
		Perpendicular to polymerisation field, $k = 2, 3$	$X_2 = X_3 = -4\left(\frac{R}{a}\right)^3 \sum_{2,3} (3)$

The fields ξ_{Fk}, ξ_{Sk} fulfil the equations [11]:

$$\xi_{Ek} = \xi_{Fk}(1-\varphi) + \xi_{Sk}\varphi \tag{9}$$

$$\chi_{Mk} = \chi_{Fk}(\xi_{Fk}/\xi_{Ek})(1-\varphi) + \chi_{Sk}(\xi_{Sk}/\xi_{Ek})\varphi \tag{10}$$

$$\chi_{Ek} = (\xi_{Ek}/\xi_k)\chi_{Mk} \tag{11}$$

where ξ_{Ek} is the effective field parameter in the sample and ξ_k is the external magnetic field parameter.

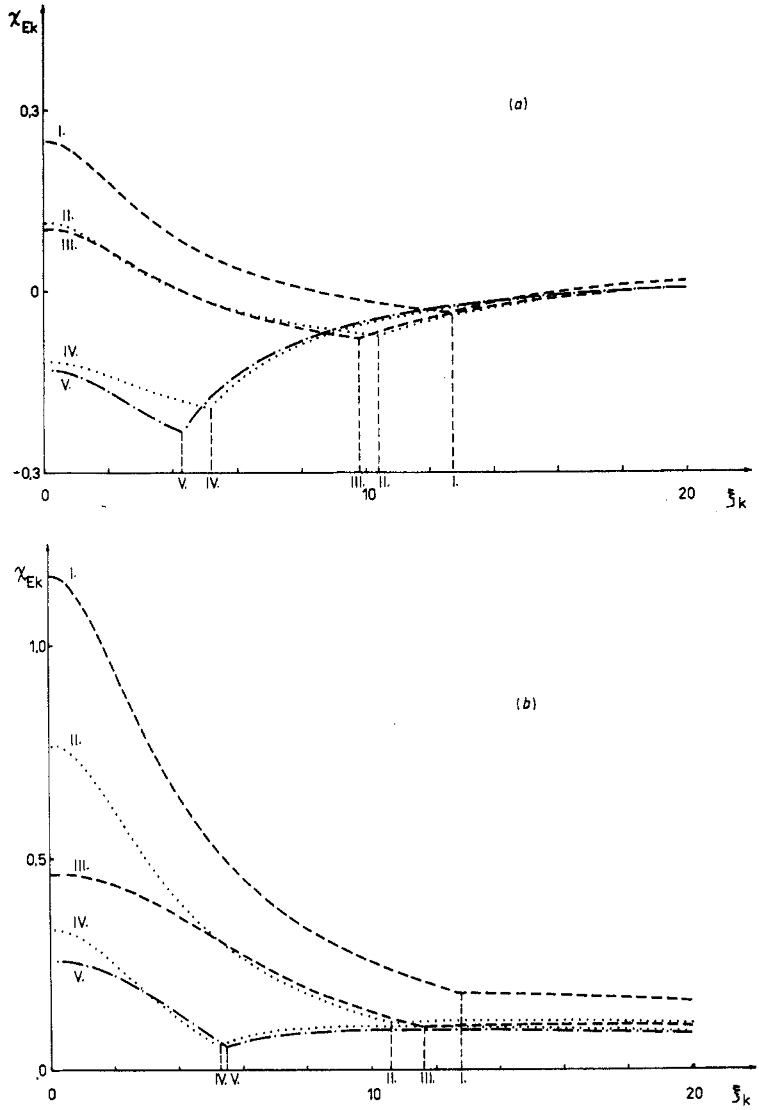

Figure 2. The susceptibility of various structures of high-T_c superconductive particles dispersed in a textured polymerised magnetic fluid versus the external magnetic fluid. The parameters used in the calculation are: $D_1 = D_2 = 0.05$, $D_3 = 0.9$, $a/R = 2.1$, $\varphi = 0.1$, $S = 0$ (for disorder), $S = 0.2$ (for texture), $\xi_{c1} = 15$ and $\xi_{c2} = 35$. (*a*) $\chi_0 = 0.3$; (*b*) $\chi_0 = 1.1$. I, II, ... V — denote various structures and various external field direction (see table 2)

From equations (8)–(11) we easily obtain the following equations

$$\xi_{Fk} = \xi_{Ek}[1 - \varphi + f_k(\varphi - X_k)]/[(1 - \varphi)(1 - X_k f_k)] \quad (12)$$

$$\xi_{Sk} = \xi_{Ek}(1 - f_k)/(1 - X_k f_k) \quad (13)$$

$$\xi_{Ek} = \xi_k \, 1/(1 + \chi_{Mk} D_k). \quad (14)$$

By means of the above-mentioned equations we have calculated the susceptibility of composites of superconductive particles in a polymerised magnetic fluid for chain and disordered structures using the following iteration algorithm:

(i) In the first step, values ξ_{Sk}, ξ_{Fk} are equal to the external field ξ_k.

(ii) According to equations (1)–(4) we have calculated the susceptibilities χ_{Fk} and χ_{Sk} respectively.

(iii) With the aid of equations (6)–(8) we obtain the effective susceptibility of the mixture.

(iv) The new values of mean fields are then calculated from equations (12)–(14). If ξ_{Sk} (new) and ξ_{Fk} (new) are equal to ξ_{Sk} (old) and ξ_{Fk} (old) the calculation of ξ_{Ek} for given value ξ_k is complete and χ_{Ek} is found.

(v) If the conditions in point (iv) are not satisfied, iteration continues. After each cycle (the cycle starts at point (ii) and finishes at point (v) or (vi) the average values of magnetic fields are calculated in the following way

$$\xi_{Fk}^{(n)} = \frac{1}{n}\sum_{i=1}^{n} \xi_{Fk}^{(i)} \qquad \xi_{Sk}^{(n)} = \frac{1}{n}\sum_{i=1}^{n} \xi_{Sk}^{(i)}$$

where index i denotes the number of the cycle and n is the number of executed cycles.

(vi) Parameters ξ_{Sk}, ξ_{Fk} are given as being equal to $\xi_{Sk}^{(n)}$ and $\xi_{Fk}^{(n)}$ respectively. The algorithm continues from point (ii).

The susceptibilities of composites of superconductive particles dispersed in a polymerised (frozen) magnetic fluid for various structures and various external field directions for two values of initial susceptibilities, χ_0, of a magnetic fluid are given in figure 2. The critical magnetic fields ξ_{c1}^* are indicated. The values of ξ_{c1}^* are given in table 2. It is shown that the critical magnetic field ξ_{c1}^* is lower than for the bulk material.

Table 2. The critical magnetic field ξ_{c1}^* for various structures and various external field directions corresponding to figure 1. χ_0 is the initial susceptibility of the magnetic fluid.

Structure	Direction of external field	(*a*) $\chi_0 = 0.3$ ξ_{c1}^*	(*b*) $\chi_0 = 1.1$ ξ_{c1}^*
I—chains	$k = 1$	12.57	12.71
II—disorder	$k = 1, 2$	10.46	10.56
III—disorder	$k = 3$	9.81	11.56
IV—chains	$k = 2$	5.17	5.24
V—chains	$k = 3$	4.31	5.49

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