

# Irreversible relaxation behaviour of a general class of magnetic systems

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**Abstract.** It is shown that magnetic systems after magnetization in a weak external field for a finite time  $t_{mag}$  exhibit a universal time-dependent relaxation behaviour. The normalized magnetization decay after switching off an external field does not depend on any sample parameters and follows a universal law  $m(t) \sim \log(1 + t_{mag}/t)$ . This universal time dependence is confirmed by magnetic relaxation measurements performed on different powders of small barium hexaferrite magnetic particles at room temperature. The measurements were performed using the PTB SQUID magnetometer in the Berlin Magnetically Shielded Room.

## 1. Introduction

A basic feature of the irreversible magnetization relaxation after a sudden change of the external field for many magnetic systems is a strongly non-exponential character of the magnetization decay  $m(t)$  [1, 2, 3]. Usually the time dependence  $m(t)$  can be fitted over many time decades quite well with a so-called linear–logarithmic law:

$$m(t) = -S \log(t/t_0) + C \quad (1)$$

where a time constant  $t_0$  is introduced to preserve the correct dimensionality and  $S$  is called the coefficient of magnetic viscosity [1]. A commonly accepted phenomenological explanation of the time dependence (1) is based on the assumptions that (i) this non-exponential magnetic relaxation is caused by thermally activated transitions between metastable and stable magnetization states and (ii) the system has a broad distribution density  $\rho(E)$  of the energy barriers  $E$  separating these states with the distribution width  $\Delta E$  much larger than the system temperature:  $\Delta E \gg T$  [2].

It can be shown [1, 2] that under these assumptions the dependence (1) can indeed be derived with the magnetic viscosity coefficient  $S = T\rho(E_c)$  proportional to the energy barrier distribution density via a so-called critical energy  $E_c = T \log(t/t_0)$ . This means that measurements of the time-dependent relaxation in magnetic systems at various temperatures can be used to obtain information about the energy barrier distribution. If the origin of this distribution is known (for example, in fine-magnetic-particle systems the size and shape distributions of particles) then this information can, in turn, be used for the determination of the corresponding system parameters [4].

Significant deviations from the linear–logarithmic dependence (1) are usually attributed to a very narrow distribution of the energy barriers. In this case the assumption  $\Delta E \gg T$

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fails, which results in more or less important corrections to the dependence (1) [5, 6]. However, deviations from this law have also been observed in systems of small magnetic particles [7, 8] where the energy barrier distribution density  $\rho(E)$  is usually broad enough to satisfy the condition  $\Delta E \gg T$ , so a new explanation for the non-linear logarithmic magnetization decay is necessary.

As far as we are aware the first time such an explanation was given was in [8, 9] where it was shown that if a system is magnetized in a weak external field for a finite time only some small fraction of metastable states are actually populated densely enough to contribute significantly to the magnetization decay measured after the removal of the external field. This leads to significant deviations from the linear–logarithmic dependence (1) for measurement times (much) larger than the magnetization time. This result was obtained in [9] in the so-called ‘critical-volume approximation’ for the special case of a system of non-interacting fine magnetic particles with a uniaxial anisotropy.

In this paper we study magnetic systems which were magnetized for some finite time  $t_{mag}$  in a weak external field (the rigorous criterion for the field strength will be derived below). We demonstrate that under very general assumptions (which will be listed below) the magnetization decay in this case follows a universal law, so the time dependence  $m(t)/m(t_{beg})$  (i.e. magnetization normalized to its value at the beginning of the measurement procedure  $t_{beg}$ ) is completely determined by the magnetization time  $t_{mag}$  only and does not depend on any other system parameters. We present experimental results confirming this universal scaling behaviour.

## 2. Theory

We consider a magnetic system where due to some intrinsic system properties a demagnetized state is highly degenerate, i.e. there exist very many microscopic system states with zero total system magnetization and with almost identical system energies. Furthermore we assume that this system can be divided into a macroscopically large number of subsystems each of which, in the absence of an external field, has at least two (meta)stable states. We will denote the magnetic moments of a subsystem in these two states as  $m_1$  and  $m_2$ , and the energy barrier between these states as  $E$ .

The simplest example of such a system is a non-interacting Heisenberg model with a uniaxial on-site anisotropy where the energy of each spin  $s$  (which is in this case one of the subsystems mentioned above) has two minima with the spin projections on the anisotropy axis  $s_{z1} = -s_{z2} = s$ . A classical analogue is the non-interacting Stoner–Wohlfarth model which describes a system of small ferromagnetic particles with a uniaxial single-particle anisotropy at a temperature much lower than the Curie temperature of the particle material. Under these conditions the magnetic moment of each particle has a constant absolute value, so the magnetization can only be reversed by coherent rotation. This also leads to two energy minima along the two opposite directions of the easy axis. This single-particle picture can also be applied to the interacting system if the single-particle anisotropy is strong enough to consider interparticle interactions as a weak perturbation.

A more complicated case is a disordered system of interacting magnetic moments (particles) with low or absent on-site (single-particle) anisotropy. In this case strong interparticle interactions in clusters of closely packed particles may lead to the cooperative particle remagnetization processes after a sudden change of the external field where mainly particles of a single cluster are involved. Hence these clusters can be considered as subsystems of the type discussed above. Here a cluster shape anisotropy leads to at least two possible equilibrium states. The general problem of whether or not in any disordered

system with long-range dipolar interactions such localized switching (remagnetization) modes dominate the remagnetization behaviour is discussed in [10].

We start with the consideration of transitions between two (meta)stable states of a subsystem (a single particle in the simplest case) with the energy barrier  $E$  between the two states. If we denote the magnetic moments in these two states as  $m_1$  and  $m_2$  and the probabilities of transitions  $1 \rightarrow 2$  and  $2 \rightarrow 1$  per unit time as  $\gamma_{12}$  and  $\gamma_{21}$  then the averaged relaxation of the  $z$ -projection of the magnetization after switching off the external field will follow an exponential law [11]:

$$m_z(t) = \delta m \Delta n_1^{(0)} e^{-\gamma(E)t} + m_z^{eq} \quad (2)$$

where  $\delta m = m_{z1} - m_{z2}$ ,  $\Delta n_1^{(0)}$  denotes the initial (i.e., immediately after the field is switched off) overpopulation of the state 1,  $m_z^{eq}$  stands for the equilibrium magnetization of the subsystem in the absence of the external field, and the relaxation speed  $\gamma(E)$  is the sum of the transition probabilities per unit time introduced above:  $\gamma = \gamma_{12} + \gamma_{21}$ .

The relative overpopulation  $\Delta n_1^{(0)}$  achieved after the system was magnetized in an external field depends on the magnitude of this field  $H$ , on the magnetization time  $t_{mag}$  and on the energy barrier  $E$ . Assuming that in the external field the first state is that with the lower energy, the application of the standard transition rate theory [11] to the transition between these states in the presence of the external field gives the relative overpopulation of the first state as

$$\Delta n_1^{(0)}(E, H, t_{mag}) = \Delta n_1^{eq}(E, H) (1 - e^{-\gamma(E, H)t_{mag}}) \quad (3)$$

where the relaxation speed  $\gamma(E, H) = \gamma_{12}(E_{12}(H)) + \gamma_{21}(E_{21}(H))$  now depends on the external field due to the field dependencies of the corresponding energy barriers  $E_{12}(H)$  and  $E_{21}(H)$ , and  $\Delta n_1^{eq}(E, H)$  denotes the difference between equilibrium populations of the first state with and without an external field:  $\Delta n_1^{eq}(E, H) = n_1^{eq}(E, H) - n_1^{eq}(E, H = 0)$ . We note that  $\Delta n_1^{eq}(E, H) > 0$  because we assumed that the energy of the first state in the external field decreases, so its equilibrium population increases.

Substituting (3) into (2) and integrating over all subsystems with different energy barriers we obtain an expression for the total time-dependent system magnetization:

$$\bar{m}_z(t) = \int_{E_{min}}^{E_{max}} dE \rho(E) \delta m(E) \Delta n_1^{eq}(E, H) (1 - e^{-\gamma(E, H)t_{mag}}) e^{-\gamma(E, H=0)t}. \quad (4)$$

Here  $\rho(E)$  denotes the energy barrier distribution density of the system, and  $E_{min}$  and  $E_{max}$  stand for the minimal and maximal energy barriers, respectively. Below we set  $E_{min} = 0$  and  $E_{max} = \infty$  which does not change the value of the corresponding integral, because outside the interval  $(E_{min}, E_{max})$  we have  $\rho(E) = 0$ . The integral of the constant term  $m_z^{eq}(E)$  present in (2) vanishes as it gives the equilibrium system magnetization, which is assumed to be zero in the absence of the external field.

Expression (4) coincides with the usual expression which describes magnetic relaxation after a system has been magnetized in a large (saturating) external field [1, 2, 4, 12] except that a factor

$$r = 1 - e^{-\gamma(E, H)t_{mag}}$$

appears. From the derivation of (4) given above it is clear that this factor accounts for the relative overpopulation of one of the two metastable states if a system was magnetized in a field of an arbitrary strength for a finite time. In standard treatments of magnetic viscosity processes when the initial magnetizing field and (or) magnetization time are assumed to be infinitely large this factor is obviously  $r = 1$ , because under such conditions only one of the two possible subsystem states is initially populated.

No further analytical progress beyond this point is possible in a general case, so some additional assumptions should be introduced. In this paper we study the case where the magnetizing field is small in a sense which is specified below. From (4) it is evident that in our case the weakness of the initial magnetic field means that its influence on the relaxation speed  $\gamma(E, H)$  corresponding to the magnetization relaxation between two (quasi)equilibrium states separated by the energy barrier  $E$  should be small, so we would be allowed to put  $\gamma(E, H) \approx \gamma(E, H = 0)$ .

To find out for which fields this would be the case, we recall that the relaxation speed depends exponentially on the energy barrier as  $\gamma = \gamma_0 \exp(-E/T)$  where the prefactor  $\gamma_0$  is supposed to be of the order  $\gamma_0 \sim 10^9 \text{ s}^{-1}$ . At this stage it is convenient to introduce a reduced relaxation speed  $\Gamma = \exp(-E/T)$  and a reduced time  $\tau = t\gamma_0$ . We also point out that due to a very large value of the prefactor  $\gamma_0$  this reduced time is  $\tau \gg 1$  for any reasonable measurement starting time (after the external field is switched off).

As was mentioned above, the relaxation speed depends on the external field due to the corresponding dependence of the energy barrier  $E(H)$ . It is again impossible to write down this dependence in a general case, but for fields much smaller than the switching field of the subsystem (which is defined as the field where one of the metastable states becomes absolutely unstable and vanishes) we can use a simple perturbation theory. Namely, if  $z$ -projections of magnetic moments of a subsystem at two metastable states and the same quantity at a saddle point corresponding to the transition between these states are  $m_{z1}$ ,  $m_{z2}$  and  $m_{sp}$  respectively, then the shift of the related energy barriers due to a small magnetic field  $H_z$  is  $E_{12(21)} \sim H_z(m_{sp} - m_{z1(2)})$ . Even if we assume that  $m_{z1} = -m_{z2}$  (which is true for a non-interacting system of fine particles with a uniaxial anisotropy but not necessarily true in a general case) so that first-order field corrections proportional to  $m_{z1}$  and  $m_{z2}$  in  $\Gamma(E, H) = \Gamma_{12}(E_{12}(H)) + \Gamma_{21}(E_{21}(H))$  cancel each other, we are still left with the correction term  $\sim H_z m_{sp}$ , which is obviously not zero because the magnetic moment of a subsystem at a saddle point can be arbitrary.

Hence the relaxation speed in a small external field can be estimated as

$$\Gamma(H) \approx \exp(-(E_0 - m_{sp}H_z)/T) = \Gamma(H = 0) \exp(m_{sp}H_z/T).$$

This means that an external field can be considered as small if  $m_{sp}H_z/T \ll 1$ . It is reasonable to expect that in a general case the  $z$ -projection of the magnetic moment at a saddle point has the same order of magnitude as the absolute value of the subsystem magnetic moment  $m_{sp} \sim m$ , which leads to the condition  $mH_z/T \ll 1$ . This inequality is not very helpful for practical purposes because it is very difficult to estimate the magnetic moment of a subsystem. Even in the simplest case of a system of particles with a large single-particle anisotropy where each particle corresponds to a subsystem we are confronted with difficulties due to the well known discrepancies between the physical and magnetic particle volumes [3]. To transform the inequality obtained above into a form which enables practical estimates of the field strength to be obtained we can use a connection between the switching field of a subsystem  $H_{sw}$  and the corresponding energy barrier  $E_0$  in the absence of the external field:  $mH_{sw} \sim E_0$ . The switching field can be estimated as the coercivity measured at very low temperatures  $H_{sw} \sim H_c(T \rightarrow 0)$ . The energy barriers which can be overcome, if we observe the relaxation at the time  $\tau$  and temperature  $T$ , are [1, 2, 5]  $E_0 \sim T \log \tau$ . Hence we obtain a final condition which should be satisfied for a magnetization field if we are going to consider this field as weak:

$$\frac{mH}{T} \sim \frac{E_0}{H_{sw}} \frac{H}{T} \sim \frac{H}{H_{sw}} \log(\gamma_0 t) \sim \frac{H}{H_c} \log(\gamma_0 t) \ll 1. \quad (5)$$

We point out that due to the large value of  $\gamma_0 \sim 10^9 \text{ s}^{-1}$  (so even for  $t \sim 10^{-3} \text{ s}$  we have  $\log(\gamma_0 t) \sim 10$ ) this condition is much stronger than the usual one  $H/H_{sw} \ll 1$ .

If the condition (5) is fulfilled we can replace  $\Gamma(E, H)$  with its value  $\Gamma(E, 0)$  in the absence of the external field and rewrite the expression for the time-dependent magnetization (4) as

$$\bar{m}_z(t) = \int_0^\infty dE f(E, H) (1 - e^{-\Gamma(E)\tau_{mag}}) e^{-\Gamma(E)\tau} \quad (6)$$

where we have collected all prefactors together in a single function  $f(E, H)$  to simplify the notation:

$$f(E, H) = \rho(E) \delta m(E) \Delta n_1^{eq}(E, H). \quad (7)$$

According to the generalized first mean-value theorem of integral calculus (one can easily verify that all conditions of this theorem are satisfied here) there exists an energy value  $E^*$  such that the integral (6) can be rewritten as

$$\bar{m}_z(t) = f(E^*, H) \int_0^\infty dE (1 - e^{-\Gamma(E)\tau_{mag}}) e^{-\Gamma(E)\tau}. \quad (8)$$

It can already be seen here that normalized (i.e., divided by its initial value) magnetization relaxation curves depend on the magnetization time and relaxation time *only* and do *not* depend on any system properties because the latter are contained in the prefactor  $f(E^*, H)$  which obviously vanishes after the normalization. To obtain this dependence we have to substitute the relaxation speed  $\Gamma = \exp(-E/T)$  into (8) and perform the corresponding integration. The result is [13]

$$\bar{m}_z(t) = T f(E^*, H) \{ \log(1 + \tau_{mag}/\tau) + \text{Ei}(-\tau) - \text{Ei}(-\tau - \tau_{mag}) \} \quad (9)$$

where  $\text{Ei}(x)$  denotes the exponential integral function

$$\text{Ei}(x) = \int_{-\infty}^x \frac{e^t}{t} dt. \quad (10)$$

In our case  $\tau \gg 1$ , so both terms containing  $\text{Ei}(x)$  are exponentially small ( $\text{Ei}(-\tau) \sim \exp(-\tau)/\tau$  etc) and can be neglected. Hence we arrive at the final expression for the time-dependent magnetization in the form

$$\bar{m}_z(t) = T f(E^*, H) \log \left( 1 + \frac{t_{mag}}{t} \right). \quad (11)$$

If we divide all measured magnetization values by the  $m_z$ -value obtained for the initial time  $t_{beg}$  when the relaxation measurements were started we obtain a universal law for the magnetization relaxation:

$$\frac{\bar{m}_z(t)}{\bar{m}_z(t_{beg})} = \frac{\log(1 + t_{mag}/t)}{\log(1 + t_{mag}/t_{beg})} \quad (12)$$

which depends neither on any properties of the particular system nor on the magnetizing field  $H$ .

This result is similar up to a normalization constant to the theoretical dependence obtained in [9] (see equation (30) in [9], for  $H \rightarrow 0$  and in our notation  $t_c \rightarrow t_{mag}$ ) and coincides with the final expression used in [9] to fit experimental data (see equation (37) in [9]), because in [9] this normalization constant is assumed to be an adjustable parameter. However, the final result in [9] is obtained for a non-interacting fine-particle system in the critical-volume approximation. This means that the width  $\Delta E$  of the energy barrier distribution density  $\rho(E)$  of the system should be much larger than the measurement

temperature:  $\Delta E/T \gg 1$  (or, to be more precise, this distribution density should vary slowly enough that its derivative  $\rho'(E) = d\rho/dE$  at the critical energy  $E_c = T \log \tau$  should be  $\rho'(E_c) \ll \rho(E_c)/T$ ). We have shown that the universal law (12) is valid for any system which (i) can be subdivided into subsystems with at least two metastable magnetization states and in which (ii) relaxation processes in different subsystems may be considered as independent at least in the main approximation. This means that the influence of the interaction between subsystems on the energy barrier height can be treated as a small perturbation (otherwise it is impossible even to write down the basic expression (4)). From the transition (6)  $\rightarrow$  (8) which uses only a general theorem of the analysis it can also be seen that *no additional restrictions* should be imposed on the distribution density of the system energy barriers  $\rho(E)$ .

In order to verify that the relaxation behaviour after magnetization in a small external field does not depend on the properties of the particular magnetic system we have performed relaxation measurements for four very different magnetic powders using the experimental procedure described below.

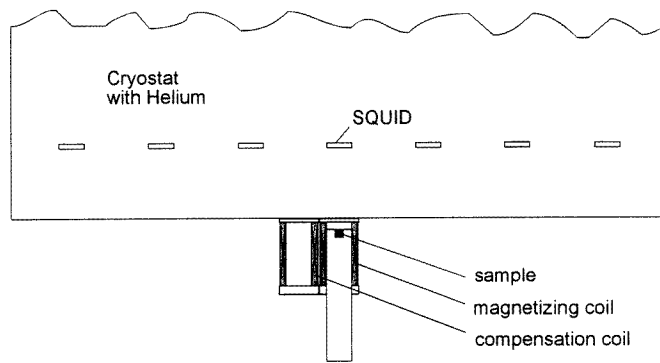
### 3. Experimental equipment and measurement procedure

The experimental measurements of the magnetic relaxation processes were performed at the laboratory for biomagnetism of the Physikalisch-Technische Bundesanstalt. Here a SQUID magnetometer is used to measure extremely weak magnetic fields (range: fT–pT) of biological origin [14]. This multichannel system features 37 SQUID sensors with a white-noise level of  $\sim 5\text{--}8$  fT Hz $^{-1/2}$  [15]. It operates in a frequency range of 0–10 kHz. However, since SQUIDs in principle cannot measure absolute field values directly, temporal variations of the magnetic field are recorded. So this system is also well suited for the study of the relaxation of magnetic systems after magnetizing in weak magnetizing fields. The sensors are arranged in three concentric rings around a central sensor at the bottom of a cryostat filled with liquid helium. In our experiments the samples were kept at room temperature and located outside the cryostat.

Measuring weak magnetic fields always involves problems with magnetic artifacts and background noise. Since urban magnetic disturbances due to e.g. power line interference or moving magnetic objects not only interfere with the measurement signal but also exceed the dynamic range of the sensors, the SQUID magnetometer is operated inside a magnetically shielded room (the Berlin Magnetically Shielded Room, BMSR), which provides a shielding factor of e.g.  $10^4$  at 0.1 Hz [15]. This enables direct measurements of fields instead of field gradients inside the chamber to be made. In order to further improve the signal-to-noise ratio a compensation technique that subtracts the weighted sum of all 37 SQUID signals from every signal value was applied. Using this special gradiometer design the component of the homogeneous field perpendicular to the plane of the sensors is reduced by a factor of typically 50 [16]. The signal of the central SQUID is chosen for further processing since it is best balanced.

On the other hand the presence of ferromagnetic material causes problems concerning the application of magnetizing fields. First of all the inner layer—especially—of the chamber must not be magnetized permanently since this would result in an unwanted residual field inside the chamber. The second problem is that if the ferromagnetic material of the chamber walls is subjected to a magnetic field it generates a time-dependent signal that interferes with the signal of the samples. Therefore a special design of the magnetizing coil has to be used.

Figure 1 shows the experimental set-up used for the measurements presented here. It



**Figure 1.** The experimental set-up used for the magnetization of samples attached to the 37-channel SQUID magnetometer.

consists of two similar solenoids of 15 mm outer diameter of 25 mm length and with 953 windings, wound in series opposition. This twin coil is fixed at the bottom of the cryostat. The right-hand coil serves as the magnetizing coil and is placed directly below the central SQUID sensor. A polystyrene tube of 10 mm diameter and 70 mm length which contains the samples fits exactly into the interior of the magnetizing coil. The left-hand coil is used to compensate the field of the right-hand (magnetizing) coil. At a distance large compared to the dimensions of the magnetizing coil the fields of the two solenoids cancel each other and thus the magnetization of the chamber walls is prevented. The direction of the magnetization of the sample is perpendicular to the SQUID plane and therefore produces a maximum signal.

Due to the geometry of the solenoids the homogeneity of the magnetizing field is poor. This results in a field constant of  $39 \text{ mT A}^{-1}$  at the centre and approximately  $10 \text{ mT A}^{-1}$  at the end of the solenoid. The timing, duration, polarity and amplitude of the current through the magnetizing twin coil are computer controlled. A maximum current of 180 mA can be applied. The sample position inside the magnetizing coil can be varied and must be taken into account for the determination of the true magnetizing field. The present set-up provides an easy and reproducible positioning of the samples and yields a maximum signal due to the measurement of the strongest field component and a minimized sample–sensor distance. An additional feature is the possibility of easily removing the samples during the measurement which enables the absolute field values to be determined.

During the application of the magnetizing field the SQUIDs must be kept at reset since the field exceeds the dynamic range of the magnetometer. As the magnetizing field is switched off the SQUID reset is released and data acquisition is started. We used a sampling rate of 1000 Hz. Signals were filtered between dc and 64 Hz. Data were collected between 5 ms and 10 s. The lower limit is determined by internal artifacts due to the pulse of the field removal. Except a baseline adjustment, no further signal processing was performed.

#### 4. Results and discussion

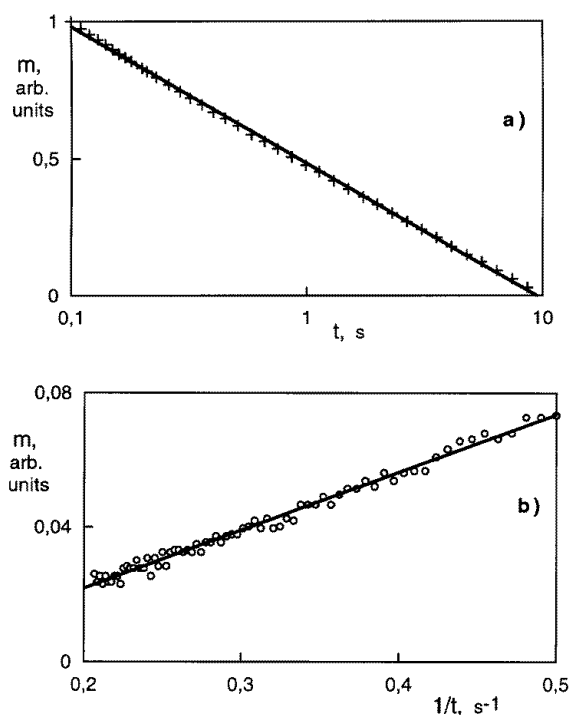
Measurements of magnetic relaxation were performed for four different magnetic powders of fine barium hexaferrite particles. The chemical composition of the particles, the mean particle size  $\bar{D}$ , the reduced remanence  $j_R$  and the coercivity  $H_c$  measured at room temperature and at  $T = 4 \text{ K}$  (for samples B and C) for these powders are collected together

in table 1. It can be seen that the powders have very different magnetic properties, primarily due to their different chemical compositions, different mean particle sizes and different size and shape distributions.

**Table 1.** Properties of various samples.

Chemical composition	$\bar{D}$ (nm)	$j_R$ (300 K)	$H_c$ (Oe) (300 K)	$H_c$ (Oe) (4 K)
A BaFe <sub>10.4</sub> Co <sub>0.8</sub> Ti <sub>0.8</sub> O <sub>19</sub>	30	0.26	560	≥ 2000
B In the form A + 3% SiO <sub>2</sub>	44	0.45	2960	3100
C BaFe <sub>10.4</sub> Co <sub>0.8</sub> Ge <sub>0.8</sub> O <sub>19</sub>	24	0.02	40	3500
D BaFe <sub>10.3</sub> Co <sub>0.85</sub> Ti <sub>0.45</sub> Sn <sub>0.40</sub> O <sub>19</sub>	26	0.20	290	≥ 2000

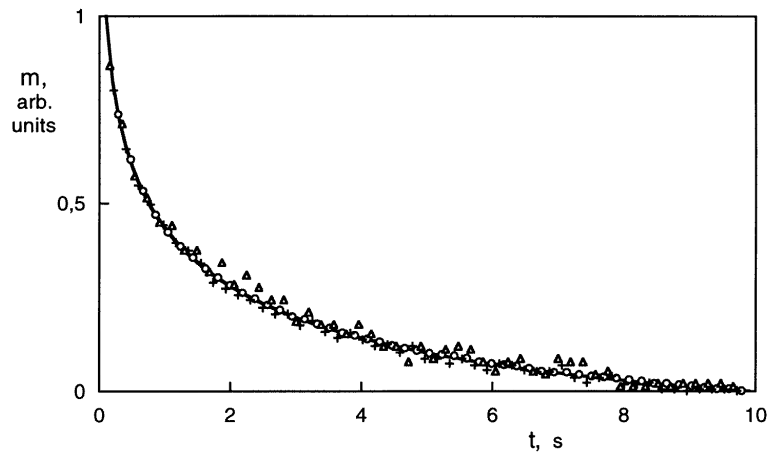
During the measurements samples were positioned near the upper end of the magnetizing solenoid; using the maximal current  $I_{max} = 180$  mA results in a field value  $H_{max} \approx 2$  mT = 20 Oe. Thus for measurement times  $t \sim 0.1$ –10 s the parameter (5)  $\alpha = (H/H_c(T \rightarrow 0)) \log(t/t_0)$  which characterizes the relative strength of the magnetic field is  $\alpha \sim (20/2 \times 10^{-3}) \log(10^{10}) \sim 0.2$ , and hence is indeed small.



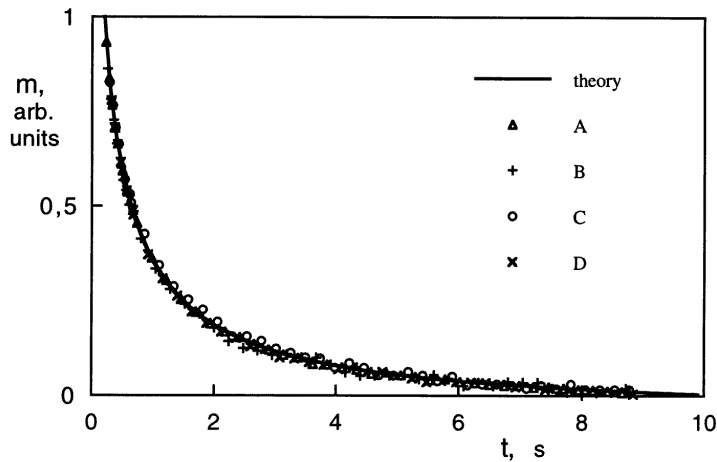
**Figure 2.** The time-dependent magnetization of the sample B for  $t_{mag} = 1000$  s (a) and  $t_{mag} = 10$  s (b) (note the different scalings of the abscissae) shown to demonstrate the linear-logarithmic dependence  $m(t) \sim \log(t)$  for  $t_{mag} \gg t$  (a) and the inverse-time dependence  $m(t) \sim 1/t$  for  $t_{mag} \ll t$  (b). The solid lines are the corresponding straight-line fits.

To test the theoretically predicted behaviour given by (12) we have performed three kinds of measurement. In the first series of experiments, magnetic relaxation curves  $m(t)$  for the





**Figure 3.** The time-dependent magnetization of sample B magnetized for  $t_{mag} = 10$  s in different magnetization fields. Different symbols show normalized  $m(t)$ -dependencies measured after the sample was magnetized with the coil currents 10 mA ( $\Delta$ ), 20 mA (+) and 100 mA (o). The solid line represents the theoretical curve (12) with  $t_{mag} = 10$  s.



**Figure 4.** Normalized magnetization relaxation curves for four different samples with the parameters given in table 1 (coil current during the magnetization: 100 mA). The experimental magnetization time  $t_{mag} = 10$  s was used for the theoretical dependence (12) (solid line).

same sample were measured after this sample was magnetized using different magnetization times  $t_{mag}$ . According to (12), for large magnetization times  $t_{mag} \gg t$  the usual linear–logarithmic magnetization dependence should be measured, whereas for small  $t_{mag} \ll t$  the dependence  $m \sim 1/t$  should be observed. These predictions are confirmed by the results shown in figure 2, where time-dependent magnetization of the sample B for  $t_{mag} = 1000$  s (figure 2(a)) and  $t_{mag} = 10$  s (figure 2(b)) are shown (note the different scalings of the abscissae). These results are analogous to corresponding results reported in [9].

The second group of experiments were carried out to verify that magnetic relaxation under the conditions given above does not depend on the magnetizing field value (up to the

initial magnetization amplitude). Time-dependent magnetization (normalized to its initial value) measured after the sample B was magnetized for  $t_{mag} = 10$  s in magnetic fields corresponding to coil currents  $I = 10, 20$  and  $100$  mA together with the corresponding theoretical dependence (12) are shown in figure 3. The only adjustable parameter when fitting the data using (12) was the signal level for  $t \rightarrow \infty$  because, as pointed out above, it is impossible to achieve the exact correspondence between zero magnetic moment of the sample and zero signal level. A good coincidence between various experimental curves and between experimental data and theoretical predictions can be seen.

Finally the independence of the properties of a particular sample predicted by (12) for the magnetic relaxation was tested. Figure 4 presents a comparison of (normalized) experimental curves measured for four samples described above (shown with different symbols) with the theoretical dependence (12) (solid line), where the magnetization value for  $t \rightarrow \infty$  was used as an adjustable parameter for the reasons described above. Again a coincidence (within experimental errors) between experimental curves for different samples and also one between experimental data and the predicted theoretical dependence can be seen. These results confirm that the time dependence of the magnetic relaxation of systems after magnetization in a weak external field for a finite time does not depend on the particular system properties. However, the initial magnetization of course depends on the particular system and can be used for its characterization.

From an experimental point of view the knowledge of the signal shape can be used for special signal-processing methods in order to detect even weak signals corrupted with a noisy background. This will help to detect even minor quantities for very dilute magnetic samples by the measurement of their relaxation signal.

The universal relaxation behaviour studied above can be used to investigate collective *delocalized* remagnetization modes of various magnetic systems, because the only assumption which is necessary for (12) to be valid is the independence of relaxation processes in different subsystems which form the magnetic system under study. This means that, for example, even fine-magnetic-particle systems with strong interparticle interactions may exhibit such a universal behaviour if these interactions lead to the well localized remagnetization modes. Further studies of this question are under way.

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