Switching Behaviour of Magnetic Particles with Dipolar Interaction *

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We study the switching in the magnetic moments of interacting magnetic particles. The dynamics of the magnetic moments is governed by a coupled set of Landau–Lifshitz–Gilbert equations. The magnetic particles are assumed to be spherical in shape, single domain, and have uniaxial anisotropy. Effects of dipolar interaction between the particles, anisotropy energy, an applied switching field with finite spatial extent and a small bias field are considered. When the separation between the particles is small, the dipolar field is significant and it affects the reversal of the magnetic moments. The final configuration attained depends sensitively on the decaying length of the switching field, the inter-particle separation, and the initial configuration. A bias field tends to suppress the effects of a spatially decaying switching field and dipolar interaction between neighbouring particles.

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As a result of rapid developments in nano-science and technology, the switching behaviour of nano-sized magnetic particles has attracted much attention recently.[1−4] Modern techniques in thin film growth can fabricate high-quality arrays of magnetic small particles,[5−8] and has led to possible designs of future high-density magnetic storage and recording devices. For devices with working mechanism depending on the dynamical response of the magnetic moments in small particles, a crucial problem is to understand the magnetization reversal or switching behaviour in magnetic small particles. In experiments, researchers used magnetic force microscopy (MFM) to investigate the influences of magnetostatic energy on the magnetization reversal behaviour of individual nanomagnets.[9−11] When the field on an MFM tip applies to a particle, its stray field may affect the states of particles nearest to the particle. Thus, how to minimize the influence of the stray field is also important for applications of MFMs.

In these nanostructures, magnetic small particles are typically arranged densely into an array. Magnetic coupling between the particles should therefore be taken into account in studying the dynamical response of the particles. Recently, we investigated the switching behaviour of a pair of coupled magnetic dipoles, with idealized switching and bias fields that act locally on only one of the dipolar moments in the pair.[12] The assumption is reasonable for widely separated particles. The switching time of the dipole moment was shown to be dependent on the orientation of neighbouring moments, and the final configuration of the dipolar pair sensitive to the separation between the two moments. However, an applied external magnetic field is not delta-function-like in space and hence does not act locally on only one particle. Instead the field decays away from the point at which it is intended to act. For densely packed magnetic particles, such a spatially decaying field affects not only a particular particle, but also the neighbouring particles, depending on the extent of the field.

In the present work, we study the effects of a spatially decaying field on the switching behaviour of coupled magnetic particles. Usually, the form of stray field produced by the MFM is very complex. For theoretical simplicity, the switching field is assumed to have an exponential decaying form centred at the particle on which the field acts. A system consisting of three coupled magnetic particles is considered, with the applied field acting on the particle in the middle. The particles are equally spaced and assumed to be identical. The Landau–Lifshitz–Gilbert[13] (LLG) equation is used to study the reversal behaviour of the three dipole moments, taking into account the effects of inter-particle dipolar interaction and anisotropy energy in the presence of a switching and a small transverse bias field. In particular, we study in detail the dependence of the final orientation of the three dipole moments on the dipolar interaction, the strength and extent of the decaying switching field, and the intrinsic anisotropic field. A small bias field is found to be useful in suppressing the effects of dipolar interaction and makes the moment easier to switch.

We consider three single-domain magnetic small particles (such a particle usually has the typical size of several nanometres to several tens of nanometres) with magnetic moments $\boldsymbol{m}_i$ $(i = 1, 2, 3)$. The particles are equally spaced on a line with separation $\boldsymbol{r}_s$ between neighbouring particles. The orientation of each magnetic moment is specified by the angles (θ_i, φ_i) of

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the magnetic moment $\boldsymbol{m}_i$, as shown in Fig. 1. The magnetic moments are treated as point dipoles. The anisotropy axes of the three particles are chosen to be parallel to each other and they lie along the z-direction. Experimentally, films consisting of array of particles with parallel anisotropy axes can be fabricated with the axes perpendicular to the film.[7] The dynamics of the three magnetic moments can be described in terms of three coupled LLG equations

$$\frac{d\boldsymbol{m}_i}{dt} = \gamma \boldsymbol{m}_i \times \boldsymbol{H}_i - \frac{\alpha}{m_i}\boldsymbol{m}_i \times \frac{d\boldsymbol{m}_i}{dt}, \quad i = 1, 2, 3 \quad (1)$$

where γ is the gyromagnetic ratio, α is a phenomenological damping constant, and $\boldsymbol{H}_i$ is the effective field acting on $\boldsymbol{m}_i$.

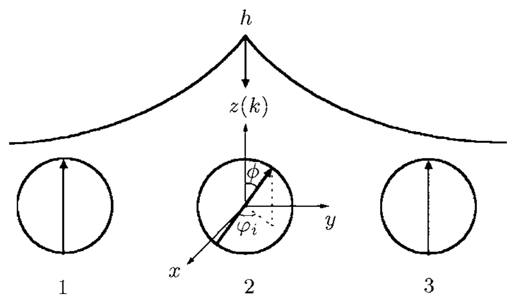

Fig. 1. Three single-domain magnetic particles modelled by three magnetic moments $\boldsymbol{m}_i$ ($i = 1, 2, 3$). The angles θ_i and φ_i specify the orientation of $\boldsymbol{m}_i$. The particles have spacing r_s between neighbouring particles. The switching field h has a finite spatial extent.

The effective field includes the external magnetic fields, the dipolar field from other particles, and the effective anisotropy field[2] given by $km_z = (2K/m^2)m_z$, where K is the anisotropy energy and m is the magnitude of the magnetic moment. For simplicity, we take $m_1 = m_2 = m_3 = m$. The effective field $\boldsymbol{H}_i$ is thus given by

$$\boldsymbol{H}_i = (h_i + km_{i,z})\boldsymbol{z} + \boldsymbol{h}_{i,int}, \quad (2)$$

where h_i is the switching field on moment $\boldsymbol{m}_i$ with $h_i = h_0 \exp(-r/r_0)$, where the decaying length r_0 sets the spatial extent to which the field decays from the particle in the middle. The field $\boldsymbol{h}_{i,int}$ acting on $\boldsymbol{m}_i$ comes from the coupling between the dipole moments which we assume to be dominated by the nearest neighbour(s). The dipolar field thus takes on the form

$$\boldsymbol{h}_{i,int} = \sum_j \frac{3(\boldsymbol{m}_j \cdot \boldsymbol{r}_s)\boldsymbol{r}_s}{r_s^5} - \frac{\boldsymbol{m}_j}{r_s^3}, \quad (3)$$

where the summation is taken over the nearest neighbouring particle(s) j of particle i, and $\boldsymbol{r}_s$ is a vector pointing from particle i to particle j. Writing $t' = tm\gamma(1+\alpha^2)^{-1}$, Eq. (1) can be rewritten as

$$m^2\frac{d\boldsymbol{m}_i}{dt'} = m\boldsymbol{m}_i \times \boldsymbol{H}_i - \alpha \boldsymbol{m}_i \times (\boldsymbol{m}_i \times \boldsymbol{H}_i), \quad i = 1, 2, 3. \quad (4)$$

More explicitly, the orientation of the moment $\boldsymbol{m}_i$ as a function of time is governed by the following equations:[12]

$$\frac{d\theta_i}{dt'} = \sin\varphi_i \frac{H_{i,x}}{m} - \cos\varphi_i \frac{H_{i,y}}{m} - \alpha\Big[-\cos\theta_i\cos\varphi_i \times \frac{H_{i,x}}{m} - \cos\theta_i \sin\varphi_i \frac{H_{i,y}}{m} + \sin\theta_i \frac{H_{i,z}}{m}\Big], \quad (5)$$

$$\frac{d\varphi_i}{dt'} = \frac{\cos\theta_i\cos\varphi_i}{\sin\theta_i}\frac{H_{i,x}}{m} + \frac{\cos\theta_i \sin\varphi_i}{\sin\theta_i}\frac{H_{i,y}}{m} - \frac{H_{i,z}}{m} - \alpha\Big[\frac{\sin\varphi_i}{\sin\theta_i}\frac{H_{i,x}}{m} - \frac{\cos\varphi_i}{\sin\theta_i}\frac{H_{i,y}}{m}\Big], \quad (6)$$

where $\boldsymbol{H}_i = (H_{i,x}, H_{i,y}, H_{i,z})$.

In general, the coupled equations (Eqs. (5) and (6)) are difficult to solve analytically. Here we solve the coupled LLG equations numerically by integrating the equations in time. Taking the magnitude m of each moment and the diameter r_p (about several nanometres to several ten of nanometres) of each particle as the units of magnetic moment and length, respectively, magnetic fields are then expressed in units of $(m \cdot r_p^{-3})$ and the gyromagnetic ratio γ in units of $(r_p^3 \cdot m^{-1} \cdot s)$.

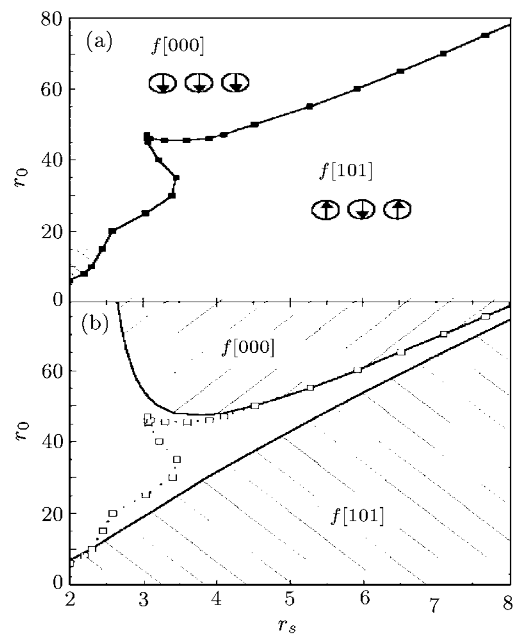

Fig. 2. Behaviour of r_0 against r_s showing the final configuration of the three moments in different regions of the plot: (a) numerical results, (b) results based on Eqs. (7) and (8). The initial condition corresponds to parallel alignments along the $+\boldsymbol{z}$-direction. Other parameters are $k/h_0 = 0.9$ and $\gamma/\alpha = 10$.

Now we consider the initial condition in which the three magnetic moments are all aligned in the z-direction. Let us denote the state aligned with the z-direction ($-z$-direction) as "1" ("0"). Therefore, the initial condition is described by the short-hand notation $i[111]$. An external switching field h is applied to flip the moment in the middle, as shown in Fig. 1. In our calculations, we choose $k/h_0 = 0.9$ so that the magnetic moment is reversed if it is an isolated particle.[2] We focus on how the separation between the particles (hence the dipolar coupling) and the decaying length of the switching field r_0 affect the final configuration of the three moments. Results obtained by solving Eqs. (5) and (6) numerically are shown in Fig. 2(a). Apart from the small shaded region, the final configuration is either $f[101]$ or $f[000]$, depending on the values of r_0 and r_s. To attain $f[101]$, i.e. to flip the central moment only, a faster decaying field with smaller r_0 should be applied when the interaction between particles becomes stronger (smaller r_s). The coupling and/or the spatial extent of the switching field tend to flip the two neighbouring moments together with the central moment, hence leading to the final configuration $f[000]$. The state $f[101]$ is easier to achieve for larger particle separations and for more localized switching field. The shaded region represents a small region in the $r_s - r_0$ parameter space in which the final configuration depends sensitively on the detailed precessions of the three moments as a function of time. These precessions of the moments, in turn, alter the instantaneous effective field acting on each of the moments. We found that in different numerical integrations of the coupled equations, either $f[101]$ or $f[000]$ is obtained, indicating that the final configuration depends not only on the *energetics* but also on the detailed *dynamics*.

To illustrate how energetic considerations can lead to a qualitative understanding of the competition between r_s and r_0, we note that the two final configurations differ only in whether the moments on the two sides are flipped or not. Therefore, we look for conditions at which the effective field in the final configuration is sufficiently strong to overcome the anisotropic field, which tends to prevent the moments from flipping. Ignoring dynamics, let us consider that the region in $r_s - r_0$ parameter space for which the final state is $f[101]$. The effective fields on the moments $\boldsymbol{m}_1$ and $\boldsymbol{m}_3$ on the two sides are

$$H_{1,3} = -h_0 e^{-r_s/r_0} + k + \frac{1}{r_s^3}. \qquad (7)$$

It follows that the final state is $f[000]$ for values of r_s and r_0 that give $H_{1,3} > 0$, i.e. with $\boldsymbol{H}_{1,3}$ pointing in the $-z$-direction. We sketch the corresponding region (shaded and labelled $f[000]$) in Fig. 2(b). Similarly, the effective fields on moments $\boldsymbol{m}_1$ and $\boldsymbol{m}_3$ for the final state $f[101]$ are

$$H_{1,3} = -h_0 e^{-r_3/r_0} + k - \frac{1}{r_s^3}. \qquad (8)$$

For values of r_s and r_0 that give $H_{1,3} > 0$, the anisotropic field keeps the moments from flipping and the final state is $f[101]$. The corresponding region (shaded and labelled $f[101]$) is also shown in Fig. 2(b). The phase boundary obtained numerically is also sketched in Fig. 2(b) for comparison. Note that the results from energetic consideration are consistent with those obtained numerically in that the region of $f[000]$ ($f[101]$) in numerical results contains the whole region of $f[000]$ ($f[101]$) followed from energetic consideration. The region between the two boundaries in Fig. 2(b) corresponds to situations in which the details of the dynamics in the precession of the moments matter.

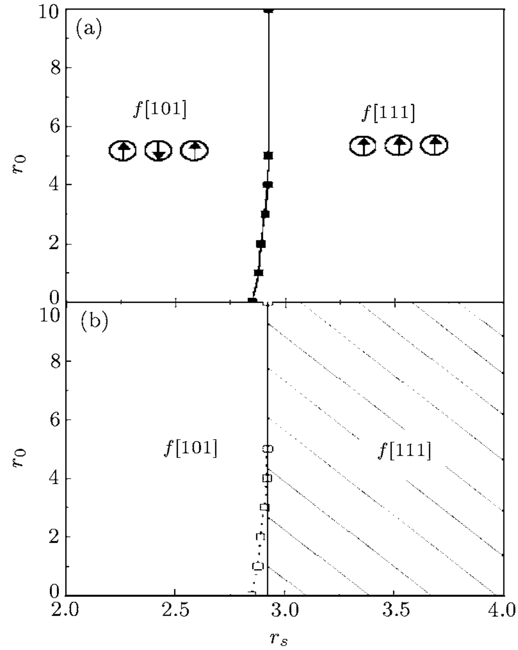

Fig. 3. A plot of r_0 against r_s showing the final configuration of the three moments in different regions of the plot. The initial condition corresponds to $\boldsymbol{m}_1$ and $\boldsymbol{m}_3$ aligned in the $+z$-direction and $\boldsymbol{m}_2$ aligned in the $-z$-direction. (a) Numerical results. (b) Results based on Eq. (9). Other parameters are $k/h_0 = 0.9$ and $\gamma/\alpha = 10$.

Similar studies can be carried out for the initial configuration $i[101]$. The switching field is now in the $+z$-direction and we look for values of r_0 and r_s that the final state is $f[111]$. Results obtained numerically by solving Eqs. (5) and (6) are shown in Fig. 3(a). The results show that it is more difficult to flip the central moment compared to $i[111]$, especially for small separations between the particles. This results from the nature of the dipolar interaction in that for particles in the x–y plane, anti-parallel alignment of neighbouring moments is energetically more favourable than parallel alignment. Thus the dipolar interaction tends to lock the configuration in [101], unless the particle separation becomes large. For the final state $f[111]$, the effective field on $\boldsymbol{m}_2$ is

$$H_2 = h_0 - k - \frac{2}{r_s^3}. \qquad (9)$$

It follows that the central moment flips for values of r_s that give $H_2 > 0$. Results are sketched in Fig. 3(b), together with the phase boundary obtained numerically. Results are in good agreement, except for a small region near the phase boundary at low r_0.

We have also carried out similar studies for the initial configurations $i[110]$ and $i[011]$, the results are similar to those for $i[111]$.

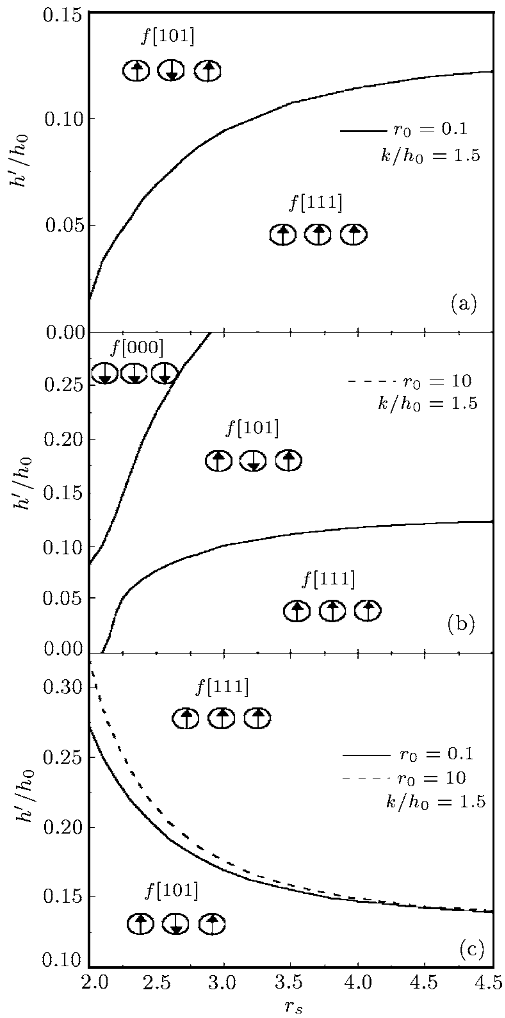

Fig. 4. Dependence of the critical value h'/h_0 of the bias field on the particle separation r_s for two different values of r_0: (a) results for rapidly decaying fields ($r_0 = 0.1$) under the initial condition of $i[111]$; (b) results for slowly decaying fields. Other parameters are $k/h_0 = 1.5$ and $\gamma/\alpha = 10$.

The results in Figs. 2 and 3 showed the non-trivial interplay between the dipolar interaction and the spatial extent of the switching field on determining the final configuration of the moments. In particular, the spatial extent could lead to switching of all the moments even when the switching field is applied to the central moment. Next we consider the effect of a small transverse bias field. Consider the situation in which the switching field is smaller than the anisotropy field ($k/h_0 > 1$), and thus it is hard to flip the magnetic moments even with the dipolar interaction considered. However, a small transverse bias field can be used to destabilize the initial configuration and lead to shorter reversal time for the cases of an isolated magnetic particle and a coupled pair of particles.[2,12] To illustrate the effect, we choose the parameters $k/h_0 = 1.5$, $h/m = 0.8$ and $\gamma/\alpha = 10$, for which the central moment cannot be flipped by the switching field. To flip the central moment, we apply a bias field in the x-direction with the same spatial decaying length to the central moment. The total external field is then given by

$$\boldsymbol{h} = h_0 e^{-r/r_0}\boldsymbol{z} - h' e^{-r/r_0}\boldsymbol{x}, \qquad (10)$$

with h' smaller than h_0. For fixed decaying length r_0, there exists a critical value of h'/h_0 above which switching is achieved for given r_s. Figures 4(a) and 4(b) show the results for the critical value of h'/h_0 as a function of r_s for two different values of r_0 under the initial condition $i[111]$. The critical value for flipping the central particle decreases with decreasing r_s, as shown in Figs. 4(a) and 4(b) for the boundary separating the $f[111]$ and $f[101]$ regions, because the dipolar interaction destabilizes the parallel alignments of the moments as separation decreases.

Rapidly decaying switching and bias fields act mainly on the moment in the middle and hence leads to the final configuration $f[101]$ for the chosen set of parameters as shown in Fig. 4(a) for the case of $r_0 = 0.1$. For slowly decaying fields, the switching field may still be significant on the two neighbouring moments. Together with a sufficiently high bias field, the neighbouring moments can also be flipped. This situation is illustrated in Fig. 4(b) for the case of $r_0 = 10$. There exists a boundary separating the regions of the final configurations $f[101]$ and $f[000]$, with the latter corresponding to all the moments flipped. Figure 4(c) shows the results for the initial configuration $i[101]$. An switching field is applied so as to attain the final configuration $f[111]$. The curves separate the regions corresponding to the final configurations $f[101]$ and $f[111]$. Due to the more stable anti-parallel alignment of moments in the initial state, a larger bias field is needed for small r_s, when the dipolar interaction is effective. Thus, the critical value of h'/h_0 decreases with increasing r_s.

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